AC Machine Drive

AC Machine

1. Induction
2. Synchronous

Note:

$$n_s = \omega_s \frac{60}{2\pi}$$

$$n_s = \frac{\omega_e}{P}\left(\frac{30}{\pi}\right)$$

$$n_s = \frac{2\pi f_e}{P} = \frac{60}{P} f_e$$

$$n_s = (120/P') f_e$$

1. Induction Machine Drive

operating principle of induction motor :-

* A 3-Φ set of voltages is applied to the stator to produce, B_s, which rotates at synchronous speed, ω_s.

$$\omega_s = \frac{\omega_e}{P}$$ where ω_e : electric radian frequency

P : Number of pole pair ($P = P'/2$)

* $\vec{B}_s$ passes over the rotor bars and induces voltages in the rotor bars :-

$$e_{ind} = (\vec{v} \times \vec{B}_s) \cdot \vec{l}$$

↳ relative velocity between rotor + $\vec{B}_s$

* e_{ind} produces a rotor magnetic field, $\vec{I}_r$.

* $\vec{I}_r$ produces $\vec{B}_r$

* $\vec{B}_r$ interacts with $\vec{B}_s$ to produce electromagnetic torque, T_{ed}.

$$\underline{T_{ed} = K \vec{B}_r \times \vec{B}_s}$$

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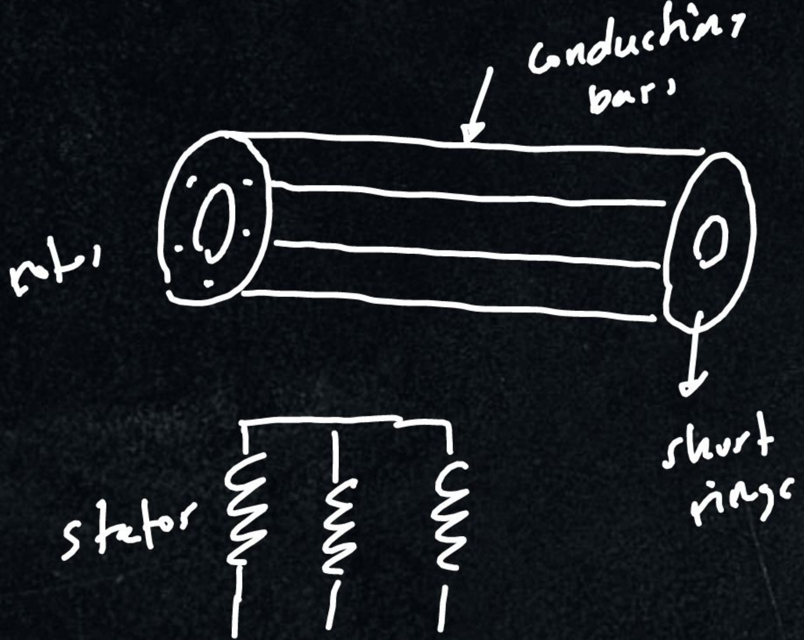
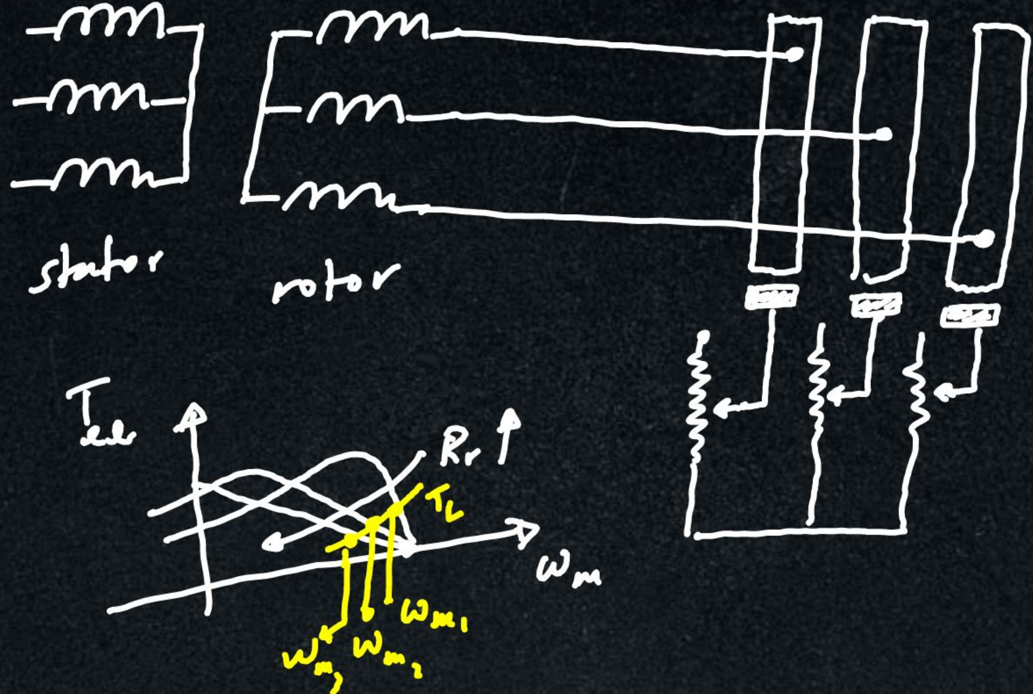
Rotor types of induction motor

Cage rotor



Wound rotor



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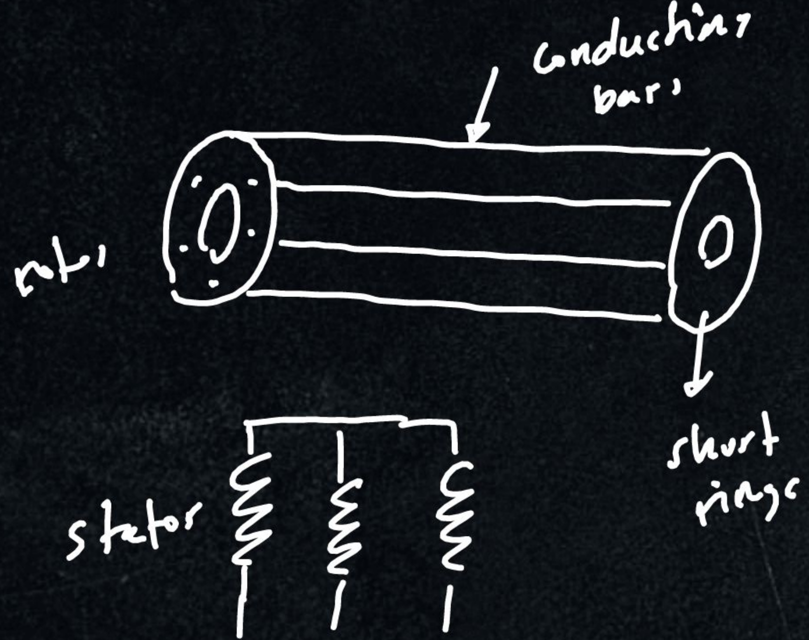
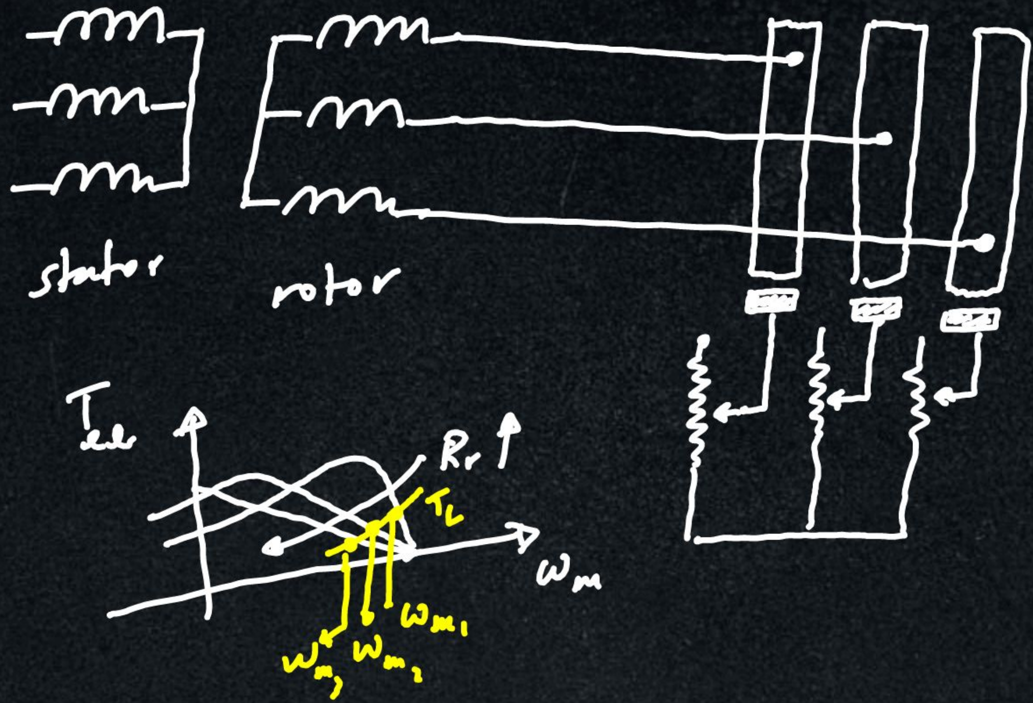
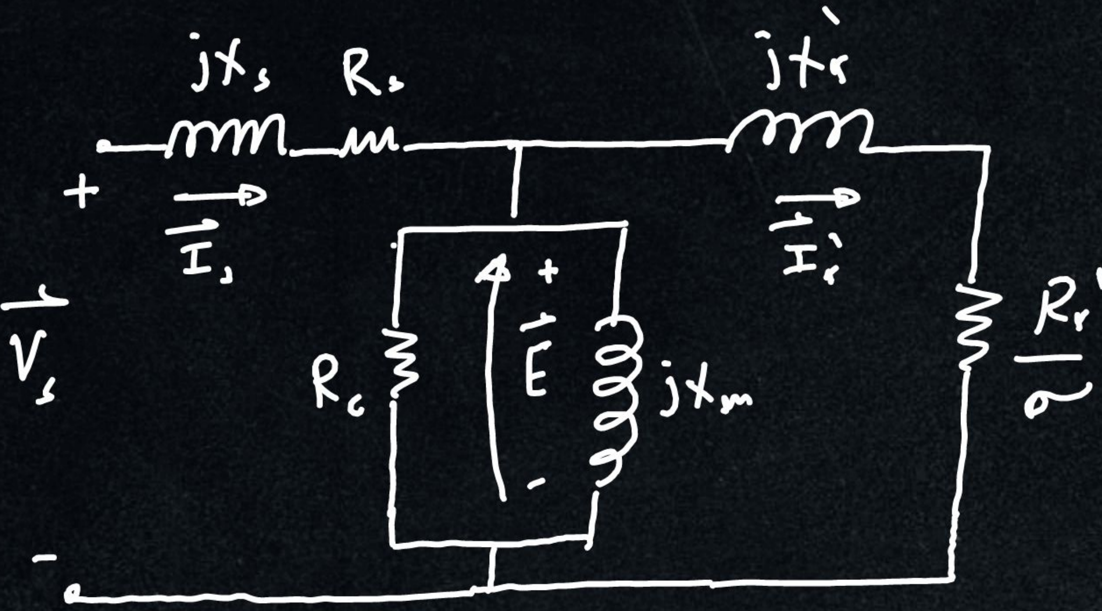


Wound rotor



Equivalent circuit



; $\underline{\sigma}$: slip

R_s : stator resistance

X_s : stator reactance ($X_s = \omega_e L_s$)

L_s : stator inductance

R_r' : rotor resistance referred to stator

X_r' : rotor reactance referred to stator

$(X_r' = \omega_e L_r')$

rotor inductance referred to stator

X_m : Magnetizing reactance

$(X_m = \omega_e L_m)$

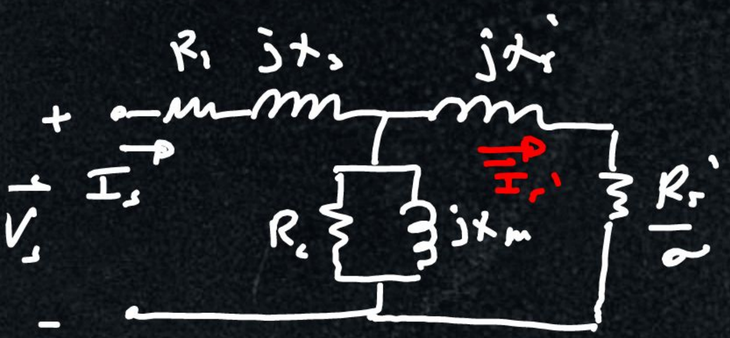
R_c : core resistance.

The concept of rotor slip :-

$$\sigma = \frac{\omega_s - \omega_m}{\omega_s} \quad ; \quad \begin{array}{l}\omega_s : \text{synchronous speed} \\ \omega_m : \text{Rotor's speed}\end{array}$$

$$\omega_\sigma = \sigma \omega_s = \omega_s - \omega_m$$

↳ slip speed



V_s : L-N voltage (rms)

$$\vec{I_s} \approx \vec{I_r'}$$

$$\Rightarrow \vec{I_r'} = \frac{\vec{V_s}}{(R_s + \frac{R_r'}{\sigma}) + j(X_s + X_r')}$$

$$I_r' = \frac{V_s}{\sqrt{(R_s + \frac{R_r'}{\sigma})^2 + (X_s + X_r')^2}}$$

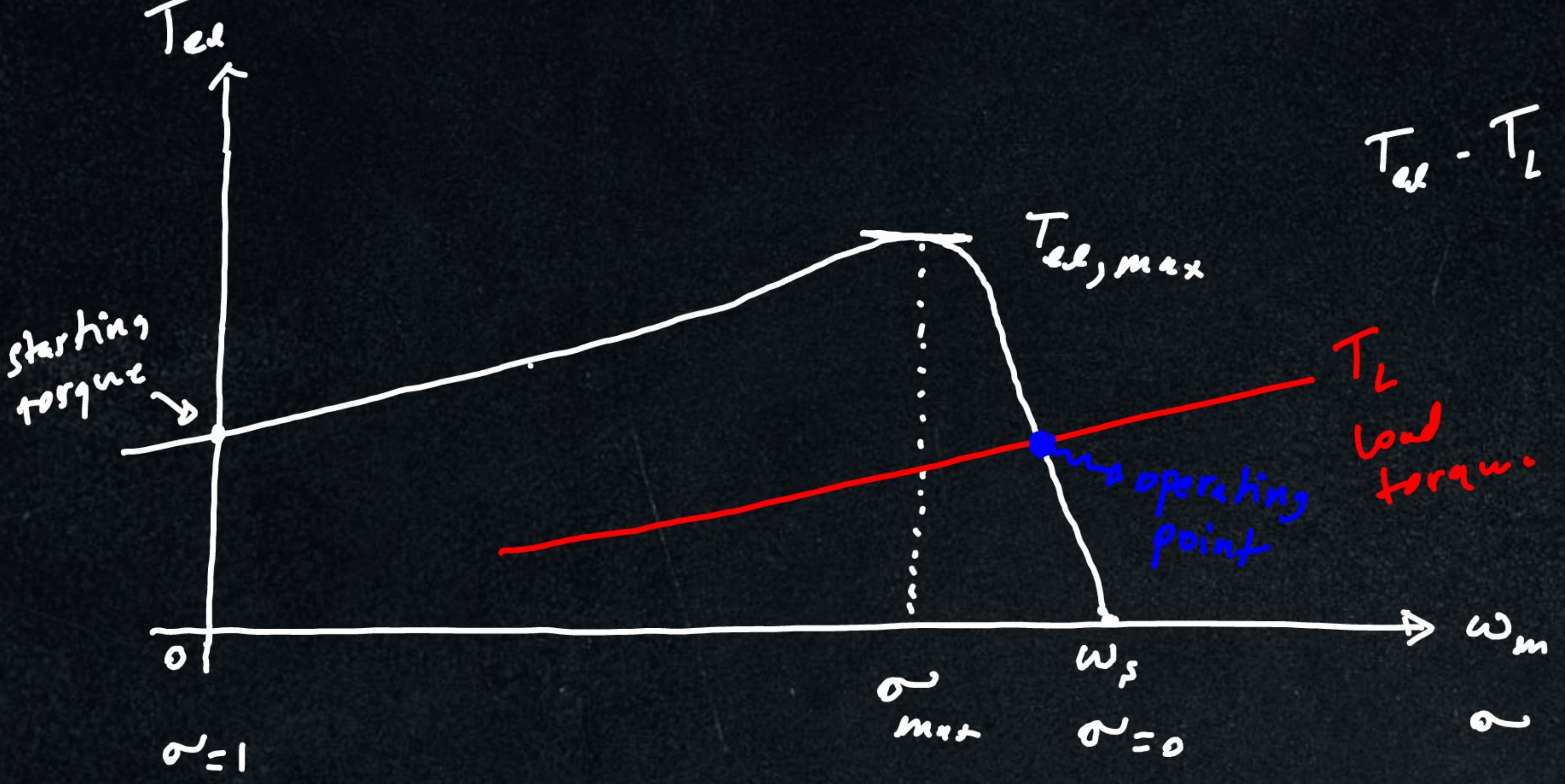
Torque equation :-

$$T_{el} = \frac{3 I_r'^2 R_r'}{\omega_\sigma} \approx \frac{3 V_s^2 (R_r'/\sigma)}{\omega_s \left[(R_s + \frac{R_r'}{\sigma})^2 + (X_s + X_r')^2 \right]}$$

Torque-speed curve

$$T_{el} = \frac{3 V_s^2 (R_r'/\sigma)}{\omega_s \left[\left(R_s + \frac{R_r'}{\sigma} \right)^2 + (X_s + X_r')^2 \right]} \quad ; \quad \sigma = \frac{\omega_s - \omega_m}{\omega_s}$$

$$T_{el} - T_L = J \frac{d\omega_m}{dt}$$



$$\sigma_{max} = \frac{R_r'}{\sqrt{R_s^2 + (X_s + X_r')^2}}$$

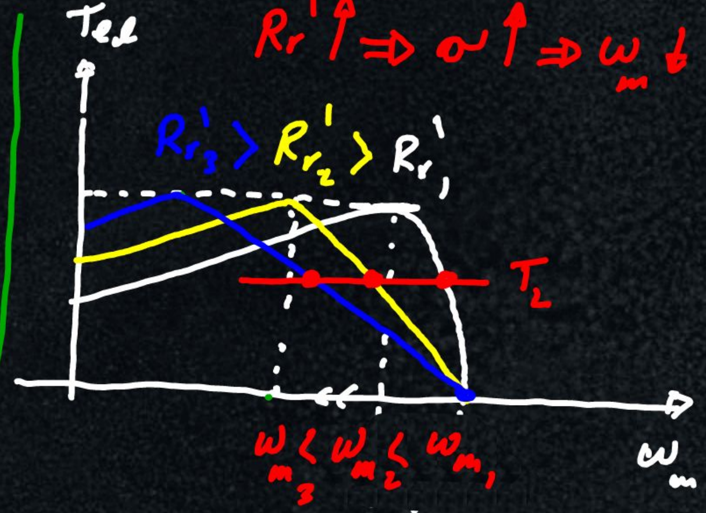
$$T_{el,max} = \frac{3V_s^2}{2\omega_s \left[R_s + \sqrt{R_s^2 + (X_s + X_r')^2} \right.} .$$

Induction motor drive - (Wound Rotor)

1) Static Rotor Resistance Control

- Recall the induced torque equation . .

$$T_{el} = \frac{3 I_r'^2 R_r'}{\omega_\sigma} = \frac{3 I_r'^2}{\omega_s} \frac{R_r'}{\sigma}$$



If $\frac{R_r'}{\sigma}$ is constant $\Rightarrow T_{el}$ is constant for a given current.

- Recall the slip equation at $T_{el,max}$

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_s^2 + X_{eq}^2}} \quad ; \quad R_r' \uparrow \Rightarrow \sigma_{max} \uparrow \qquad T_{el,max} = \frac{3 V_s^2}{2\omega_s \left[R_s + \sqrt{R_s^2 + X_{eq}^2} \right]}$$

Note :-

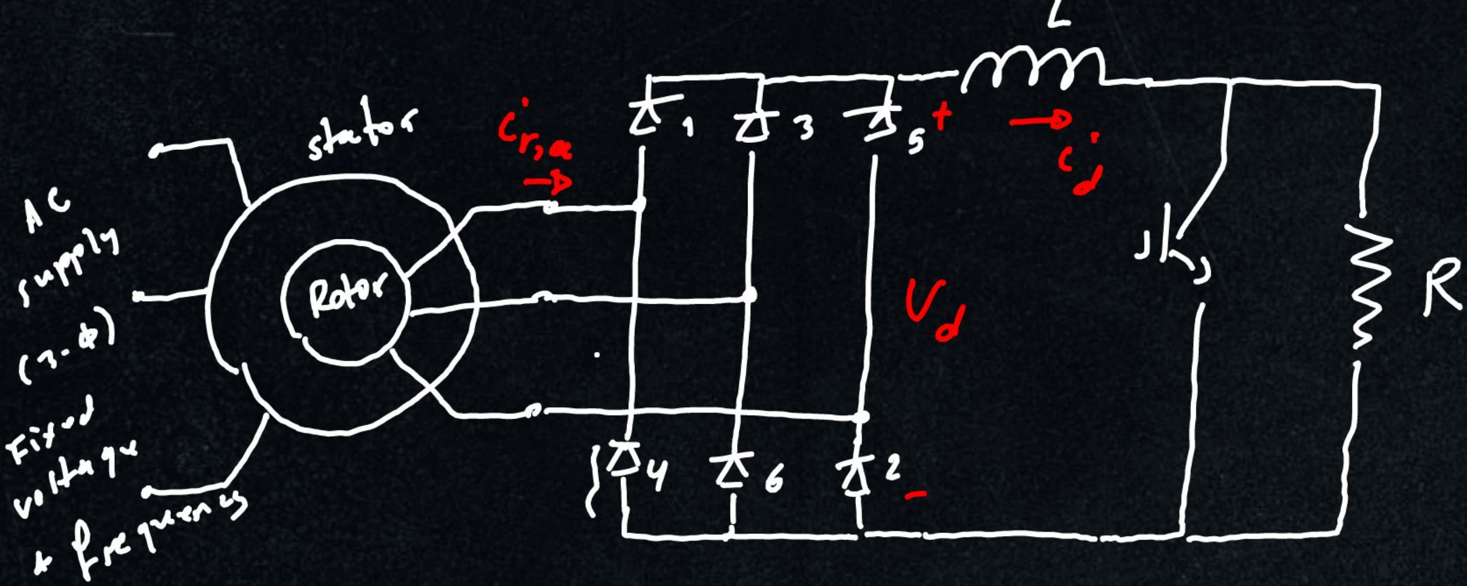
$R_r' \uparrow \Rightarrow \sigma \uparrow \Rightarrow$ Rotor's efficiency $= (1-\sigma) \downarrow$

$\Rightarrow$ Thus, this method is inefficient method of speed control. However, it has an advantage of constant torque operation.

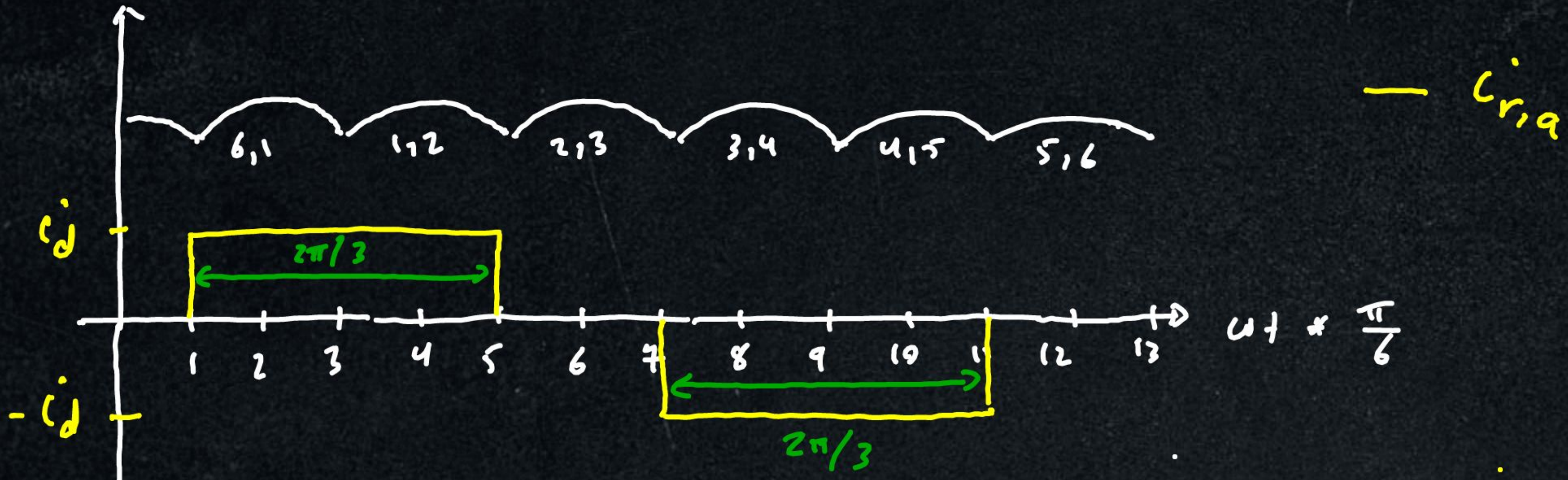
* The static rotor resistance control is implemented using diode bridge and a chopper

static rotor resistance control

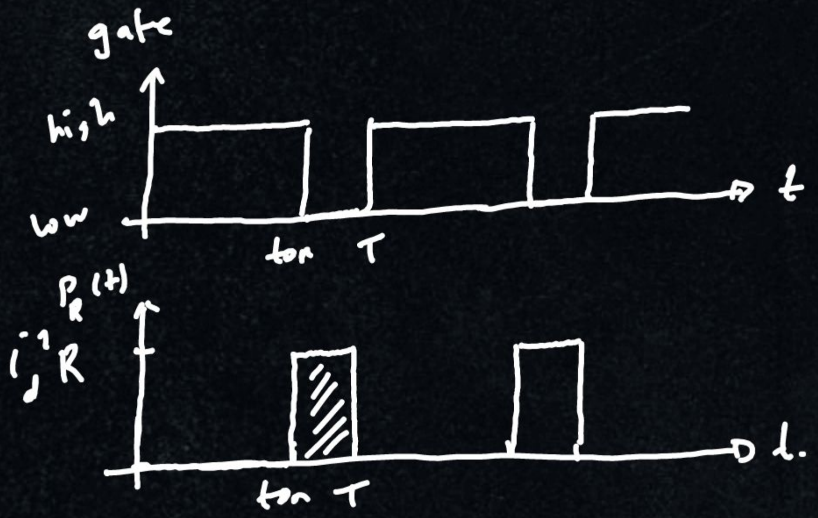
"Drive circuit"

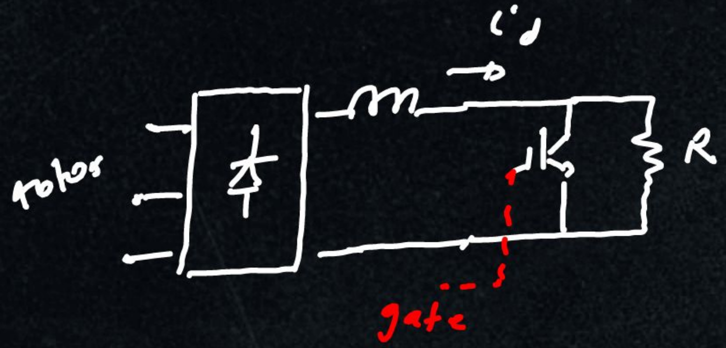


i_d: Filtered current "constant"



Equivalent circuit





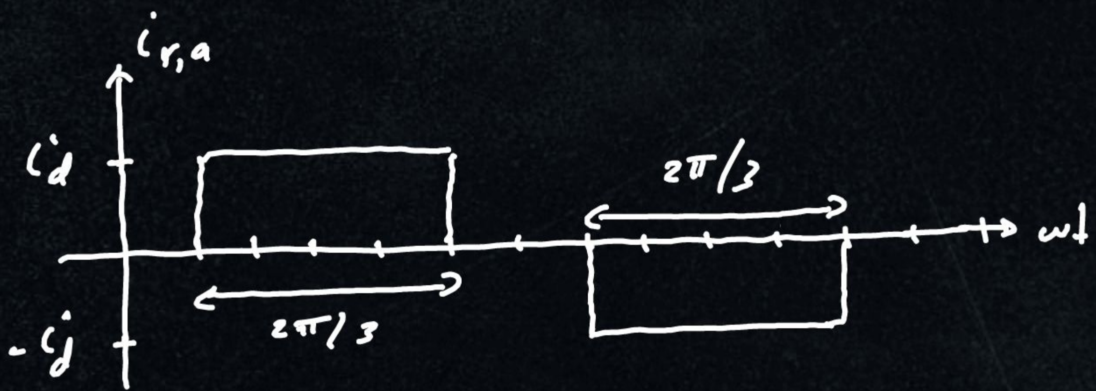
$P_R(t)$: instantaneous power absorbed by R.

The average power absorbed by R.

$$P_R = \frac{1}{T}\int_0^T P_R(t)\,dt = \frac{T - t_{on}}{T}\, i_d^2 R = (1-\delta)\, i_d^2 R$$

$$P_R = i_d^2 R^* \; ; \; R^* = (1-\delta) R$$

R^* : Effective value of R



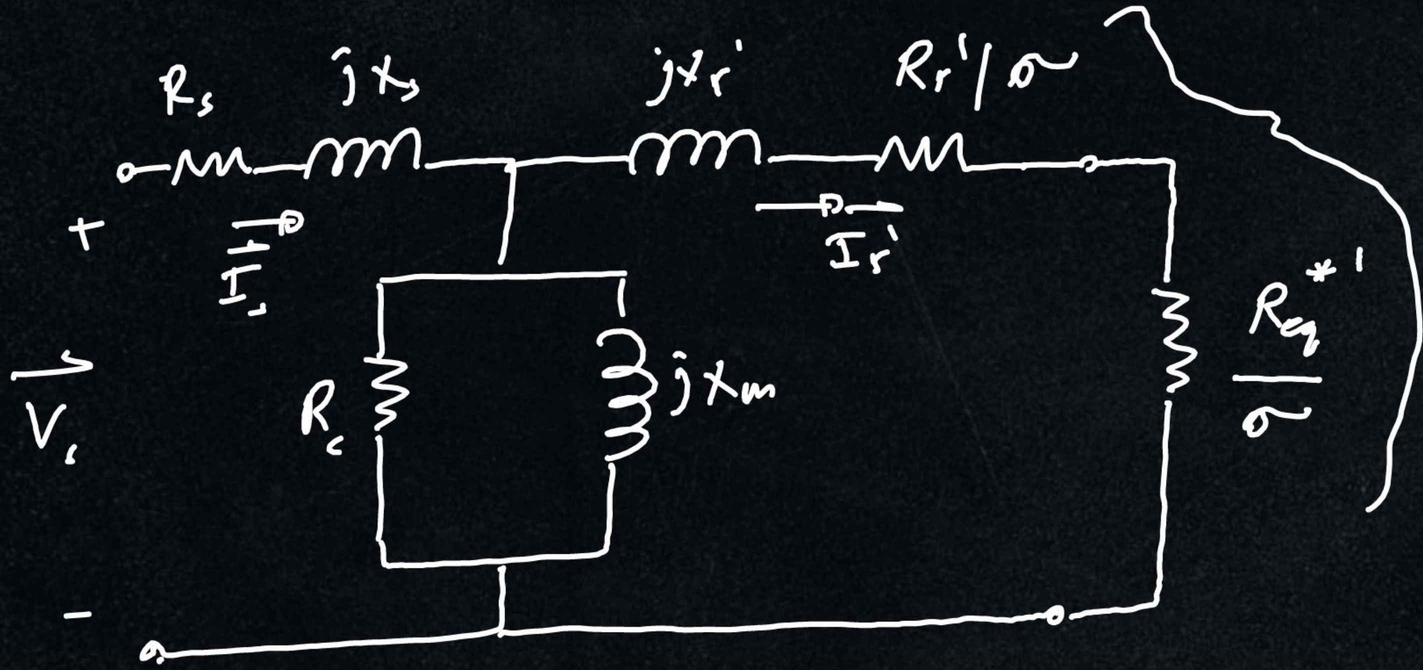
The RMS value of $i_{r,a}$:-

$$I_r = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_{r,a}^2 \, d(\omega t)} = \sqrt{\frac{2}{2\pi} i_d^2 \left(\frac{2\pi}{3}\right)} = \sqrt{\frac{2}{3}}\, i_d$$

→ R^* is converted into 3-ϕ equivalent resistor in the rotor circuit by equating their losses as follows

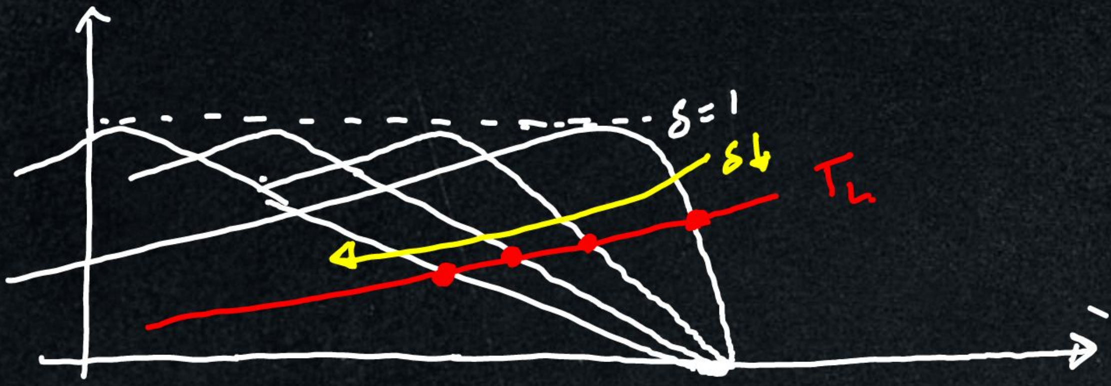
$$3 I_r^2 R_{eq}^* = i_d^2 R^* \quad ; \quad R^* = (1-s)R$$

$$\cancel{3}\left(\frac{2}{\cancel{3}}\right)\cancel{i_d^2} R_{eq}^* = \cancel{i_d^2} R^* \Rightarrow \boxed{R_{eq}^* = \frac{1}{2} R^*} \quad ; \quad R_{eq}^* = \frac{1}{2}(1-s)R$$



$$\frac{R'_{r,new}}{\sigma}$$

$$R_{eq}^{*\prime} = a^2 R_{eq}^{*} = a^2 \tfrac{1}{2}(1-s)\,R$$



Control System

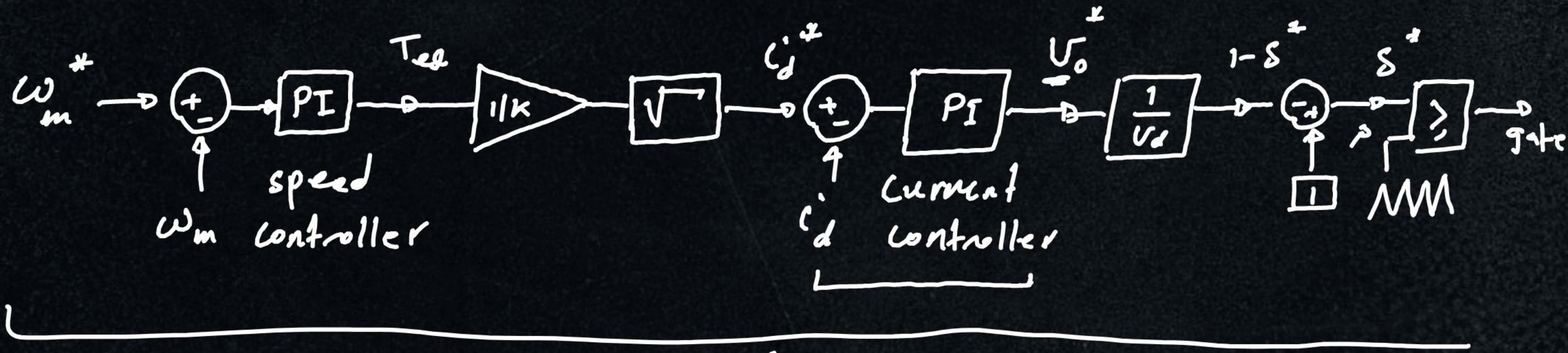
$$T_{el} = \frac{3 I_r'^2 R_r'}{\omega_s \sigma}$$

$\frac{R_r'}{\sigma}$: constant , ω_s = constant

$$I_r' = \frac{1}{a} I_r \; ; \; I_r = \sqrt{\frac{2}{3}}\, i_d$$

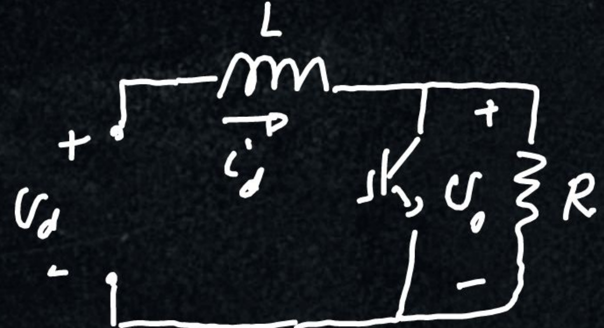
$$I_r' = \sqrt{\frac{2}{3}}\, \frac{1}{a}\, i_d$$

$$T_{el} = \underbrace{\frac{3}{\omega_s} \frac{R_r'}{\sigma} \frac{2}{3} \frac{1}{a^2}}_{\text{constant}}\, i_d^{\,2} \Rightarrow \underline{T_{el} = K\, i_d^{\,2}}$$



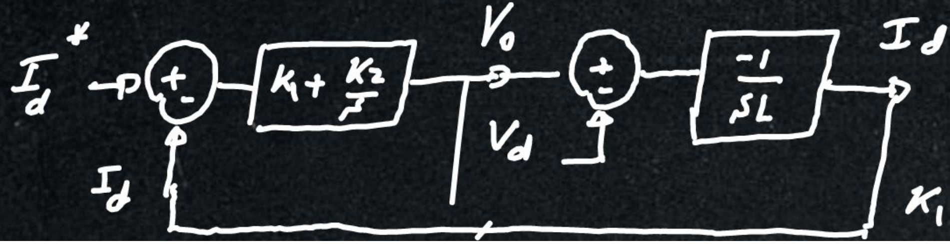
control system.

Design of current controller



$$V_d = L\frac{di_d}{dt} + V_o \Rightarrow V_o = V_d - L\frac{di_d}{dt}$$

$$\mathcal{L} \rightarrow V_o = V_d - sL\,I_d$$



$$\left.\frac{I_d}{I_d^*}\right|_{V_d=0} = ??$$

$$K_1 = -2\zeta\omega_n L \,, \quad K_2 = -\omega_n^2 L$$

EX :- A 4 pole, 3 hp, 415 V, 50 Hz, Y-connected, 3-ϕ induction motor has the following parameters per phase referred to the stator side :-

$$R_s = R_r' = 0.8\ \Omega, \quad X_s = X_r' = 3.5\ \Omega, \quad a = 2.5$$

Friction and windage losses = 170 W

(a) Calculate the slip at full load.

$$\omega_s = \frac{\omega_e}{P} = \frac{2\pi \times 50}{2} = 50\pi \text{ rad/sec}$$

$$T_{el,r} = \frac{3V_s^2}{\omega_s\left[\left(\frac{R_r'}{\sigma_r} + R_s\right)^2 + (X_s + X_r')^2\right]} \cdot \frac{R_r'}{\sigma_r} \quad ; \quad V_s = \frac{415}{\sqrt{3}}\ \text{V}$$

$$\omega_s = 50\pi \text{ r/sec.}$$

$$T_{el,r} = \frac{P_{AG}}{\omega_m} \quad ; \quad P_{AG} = 3 \times 746 + 170 = 2408\ W$$

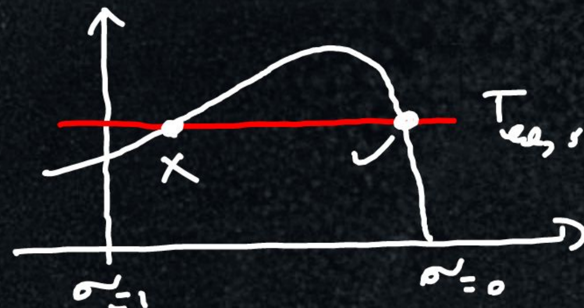
$$T_{e,r} = \frac{2408}{\omega_m} = \frac{3 V_s^2 (R_r' / \sigma_r)}{\omega_s \left[\left(R_s + \frac{R_r'}{\sigma_r} \right)^2 + X_{eq}^2 \right]}$$

3 hp

$P_m = T_L \omega_m$

P_{out}

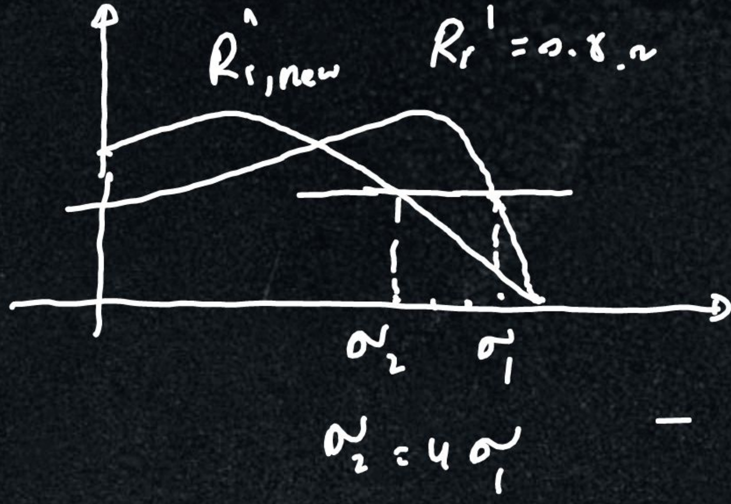
$$\sigma = \frac{\omega_s - \omega_m}{\omega_s} = 1 - \frac{\omega_m}{\omega_s} \Rightarrow \omega_m = (1 - \sigma)\,\omega_s$$



$$\frac{2408}{(1-\sigma_r)\cancel{\omega_s}} = \frac{3 V_s^2 \frac{R_r'}{\sigma_r}}{\cancel{\omega_s} \left[\left(R_s + \frac{R_r'}{\sigma_r} \right)^2 + X_{eq}^2 \right]}$$

$$\frac{2408}{(1-\sigma_r)} = \frac{3 \left(\frac{415}{\sqrt{3}} \right)^2 \left(\frac{0.8}{\sigma_r} \right)}{\left(\frac{0.8}{\sigma_r} + 0.8 \right)^2 + 7^2} \Rightarrow \boxed{\sigma_r = 0.0011}$$

(b) If the rotor is star connected, determine the external resistance inserted in series with the rotor windings such that the slip would increase to four time the value obtained in (a) with full load torque remains constant.



$$\frac{R'_r}{\omega} = \text{constant}$$

$$\frac{0.8}{0.0011} = \frac{R'_{r,new}}{4(0.0011)} \Rightarrow R'_{r,new} = 3.2\,\Omega$$

$$R'_{r,new} = R'_r + R'_{ext} = 0.8 + R'_{ext} = 3.2$$

$$R'_{ext} = 2.4\,\Omega . \quad R_{ext} = \frac{2.4}{a^2} = \frac{2.4}{(2.5)^2}\,\Omega$$

② Static Kramer Drive "Slip Energy Recovery Scheme"

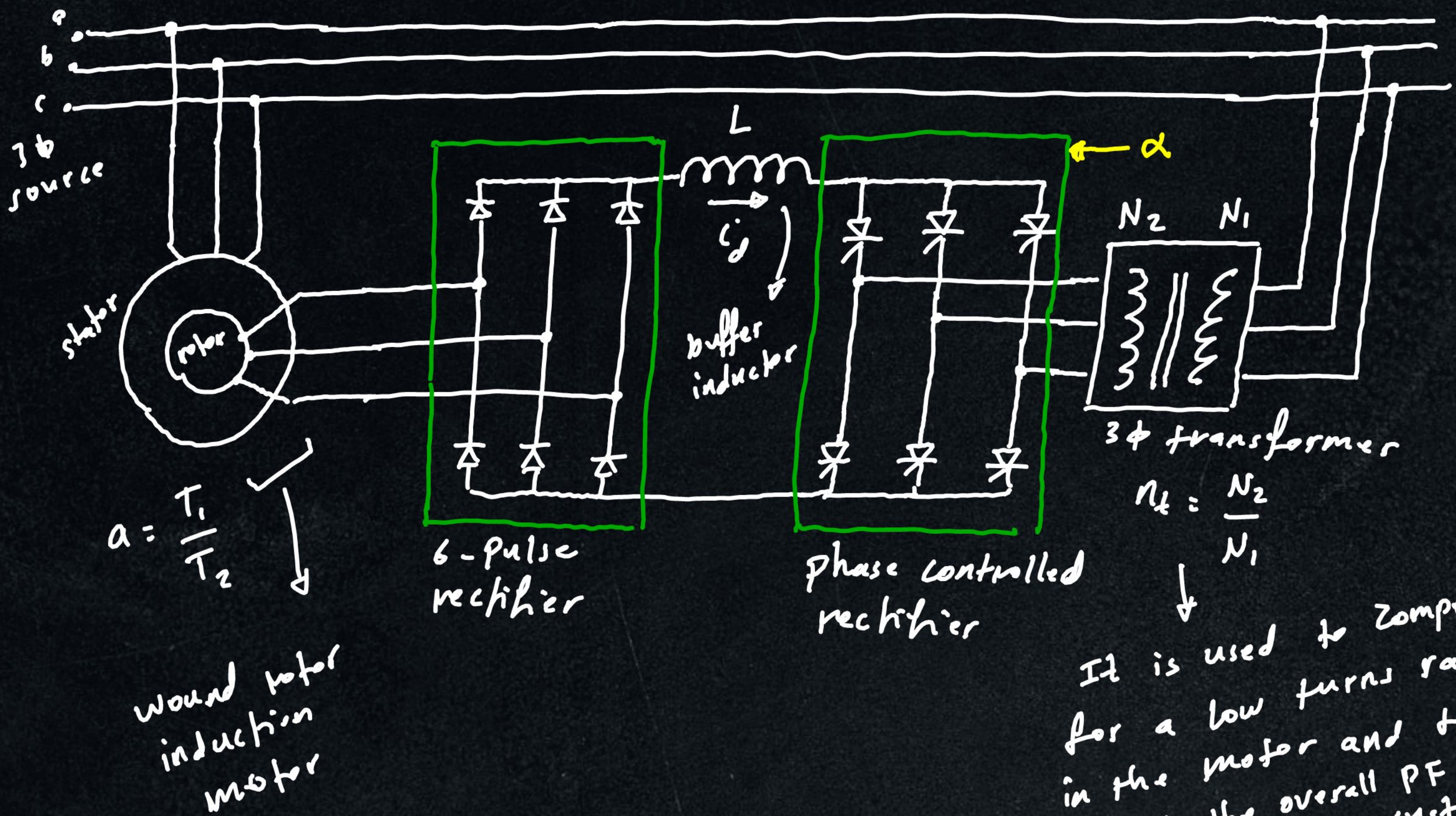
- The rotor's efficiency is reduced when a static rotor resistance drive circuit is used. Some part of slip power is lost as I^2R.

- This slip power can be recovered and supplied back to the source to improve the overall efficiency of the motor.

- Slip power $= P_{\sigma} = \sigma P_a$; $P_a = \dfrac{3 I_r'^2 R_r'}{\sigma}$

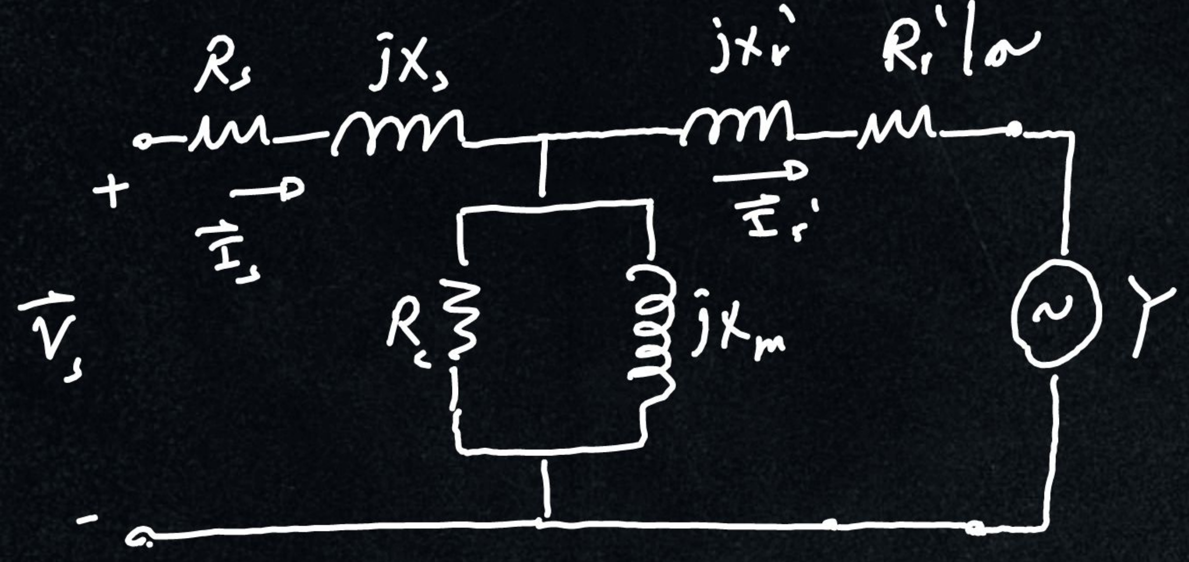
- This can be done by replacing the DC chopper and R by a 3-ϕ controlled rectifier.

- Structure of static Kramer drive



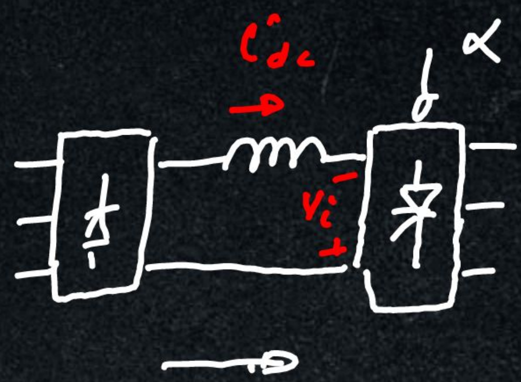
It is used to compensate for a low turns ratio in the motor and to improve the overall PF of the system.

Equivalent circuit of motor with static Kramer drive

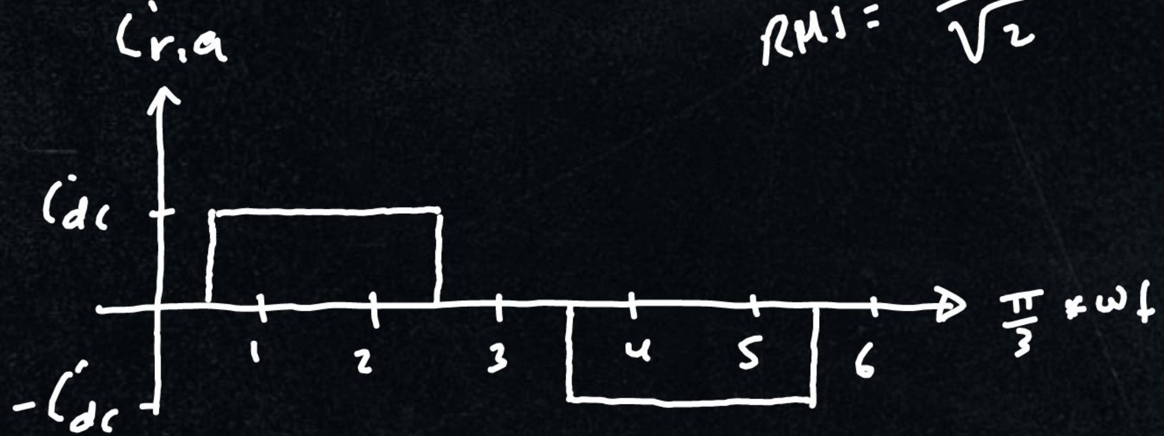


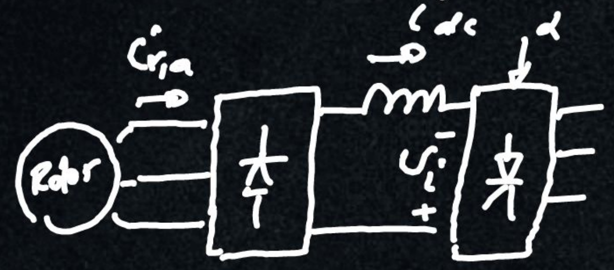
The power transferred through the controlled rectifier is given by :

$$P_i = i_{dc} V_i = 3 I_r' (Y)$$



$RMS = \frac{Peak}{\sqrt{2}}$





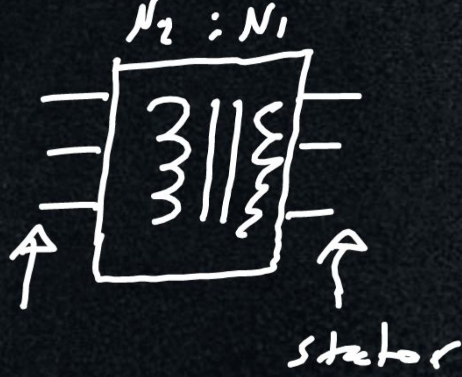
$P_i = i_{dc} V_i$

$$i_r = \sum_{i=1}^{\infty} \sqrt{2}\, \underbrace{I_{r,i}}_{RMS} \cos(i\omega t + \theta_i)$$ Fourier series

$I_{r,1} = \frac{\sqrt{6}}{\pi} i_{dc}$ → RMS value of the main component.

$$\boxed{i_{dc} = \frac{\pi}{\sqrt{6}} I_{r,1}}$$

$$V_i = \frac{3\sqrt{3}\, V_{p,\,transformer}}{\pi} \cos\alpha$$



$$V_i = \frac{3\sqrt{3}\, V_{p,\,stator}}{\pi} \eta_t \cos\alpha$$

$$\eta_t = \frac{N_2}{N_1}$$

$V_{p,\,stator} = \sqrt{2}\, V_s$, where V_s is L-N voltage (RMS)

$$\boxed{V_i = \frac{3\sqrt{6}}{\pi} V_s\, \eta_t \cos\alpha}$$

$$P_i = i_{dc} V_i$$

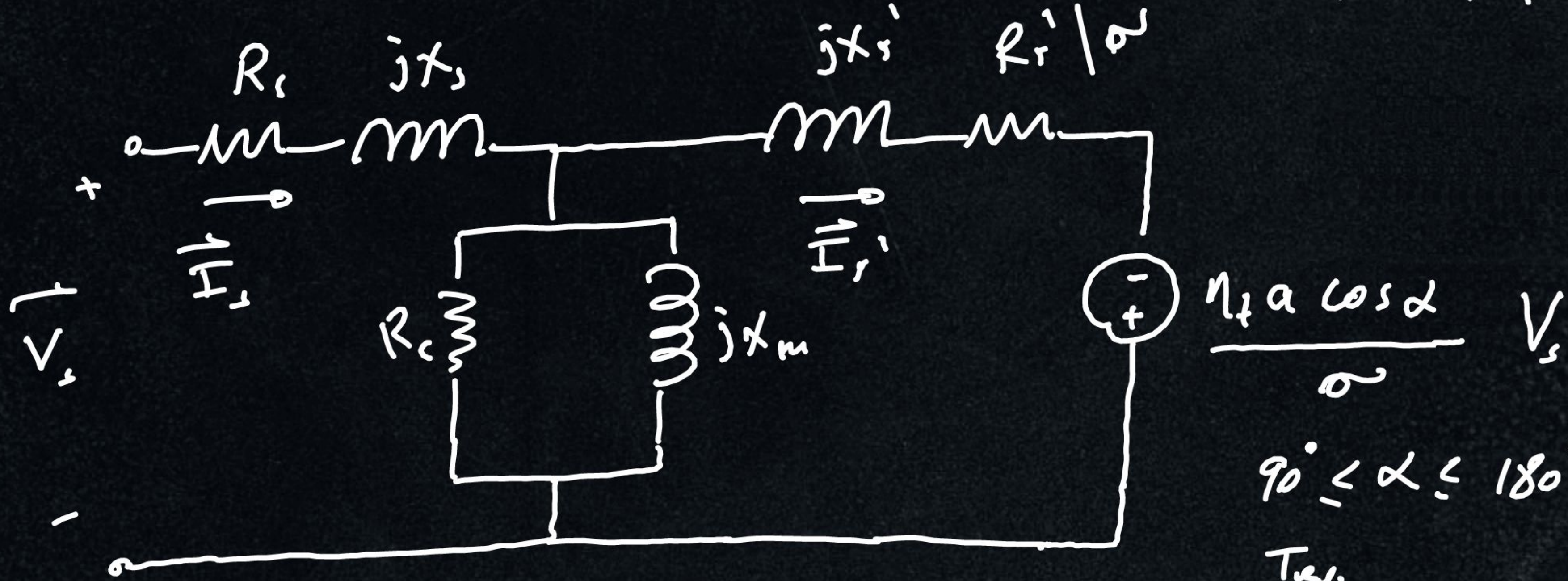
$$i_{dc} = \frac{\pi}{\sqrt{6}} I_{r_1}$$

$$V_i = \frac{3\sqrt{6}}{\pi} n_t \cos\alpha \, V_s$$

$$P_i = 3 \left(n_t \cos\alpha \, V_s \right) I_{r_1}$$

Final equivalent circuit

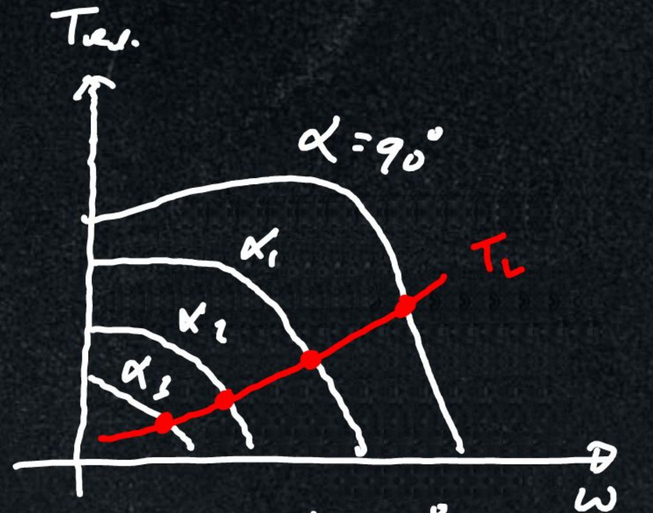
$$P_i = 3\,(\eta_t V_s \cos\alpha)\, I_r$$



$$90^\circ \le \alpha \le 180^\circ$$

Torque equation

$$T_{el} = \frac{3}{\omega_s} I_r'^{2} \frac{R_r'}{\sigma} - \frac{3\,\eta_t\, a \cos\alpha}{\omega_s\, \sigma} V_s\, I_r'$$



$$180^\circ > \alpha_3 > \alpha_2 > \alpha_1 > 90^\circ$$

$$T_{el} = \frac{3I_{r_1}'}{\omega_s}\left[\underbrace{I_{r_1}'\frac{R_r'}{\sigma}} - \underbrace{\frac{a\,n_t\cos\alpha}{\sigma}V_s}\right]$$

The torque can be approximated as

$$T_{el} \approx \frac{3V_s I_{r_1}'}{\omega_s}$$

This is valid by neglecting X_r', X_s, R_1 and excitation branch.

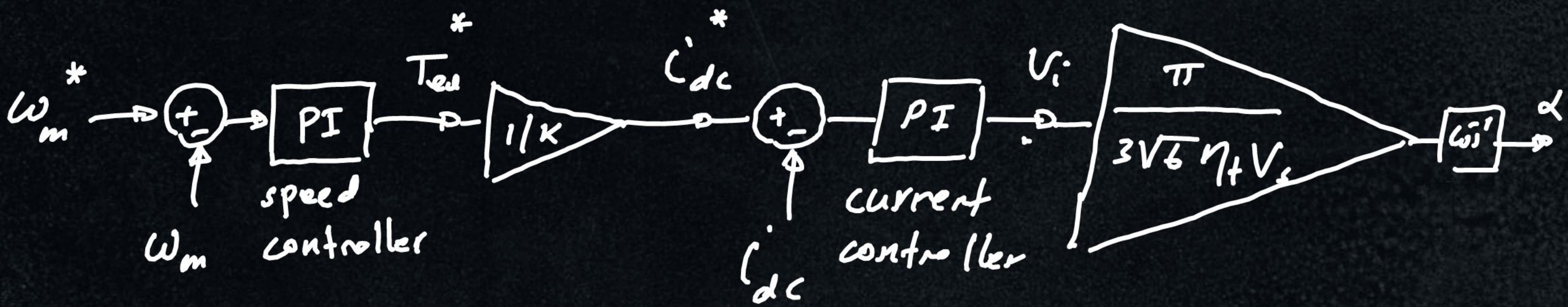
$$I_{r_1}' = \frac{I_{r_1}}{a} \quad ; \quad I_{r_1} = \frac{\sqrt{6}}{\pi} i_{dc}.$$

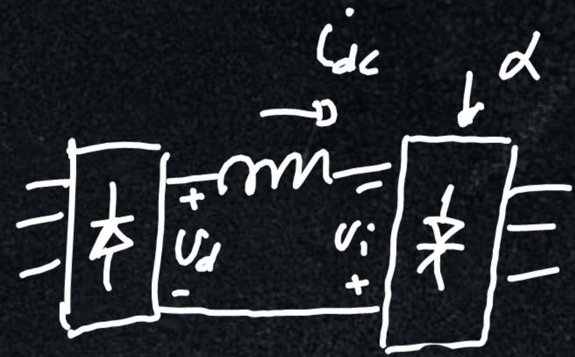
$$I_{r_1}' = \frac{\sqrt{6}}{a\pi} i_{dc} \Rightarrow T_{el} \approx \frac{3V_s}{\omega_s}\frac{\sqrt{6}}{a\pi} i_{dc} = K i_{dc}$$

$$T_{el} = K i_{dc} \quad ; \quad K = \frac{3\sqrt{6}}{\pi}\frac{V_s}{a\omega_s}$$

control system

$$V_i = \underbrace{\frac{3\sqrt{6}\,\eta_t V_s}{\pi}}\cos\alpha$$





Design of current controller :-

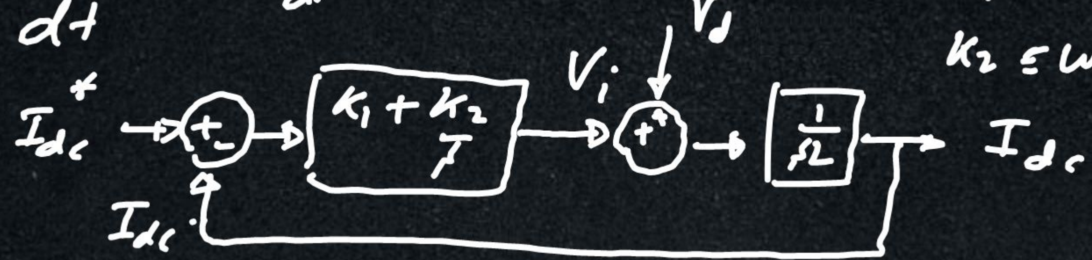
$$V_d = \frac{L\,di_{dc}}{dt} - V_i \Rightarrow V_i = \frac{L\,di_{dc}}{dt} - V_d$$

$\mathcal{L}$

$$\Rightarrow V_i = sL\,I_{dc} - V_d$$

$K_1 = 2\zeta\omega_n L$

$K_2 = \omega_n^2 L$



Induction Motor Drive - Cage rotor

① stator voltage control

- The torque developed by motor:-

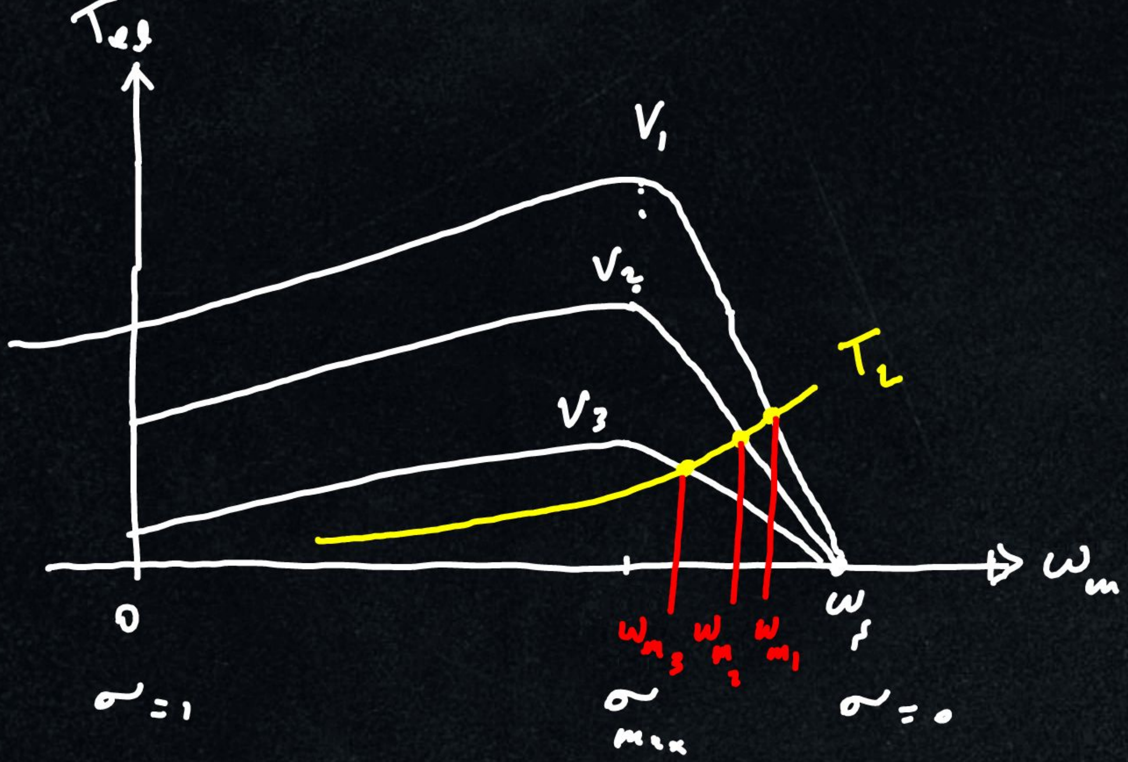
$$T_{el} = \frac{3 I_r'^2 R_r'}{\omega_m} \approx \frac{3 V_s^2 (R_r'/\sigma)}{\omega_s \left[(R_s + R_r'/\sigma)^2 + X_{eq}^2 \right]} \Rightarrow T_{el} \propto V_s^2$$

- Maximum torque

$$T_{el,max} = \frac{3 V_s^2}{2 \omega_s \left[R_s + \sqrt{R_s^2 + X_{eq}^2} \right]} \Rightarrow T_{el,max} \propto V_s^2 .$$

- Maximum slip

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_s^2 + X_{eq}^2}} = \text{constant}$$



$\omega_{m_3} < \omega_{m_2} < \omega_{m_1}$

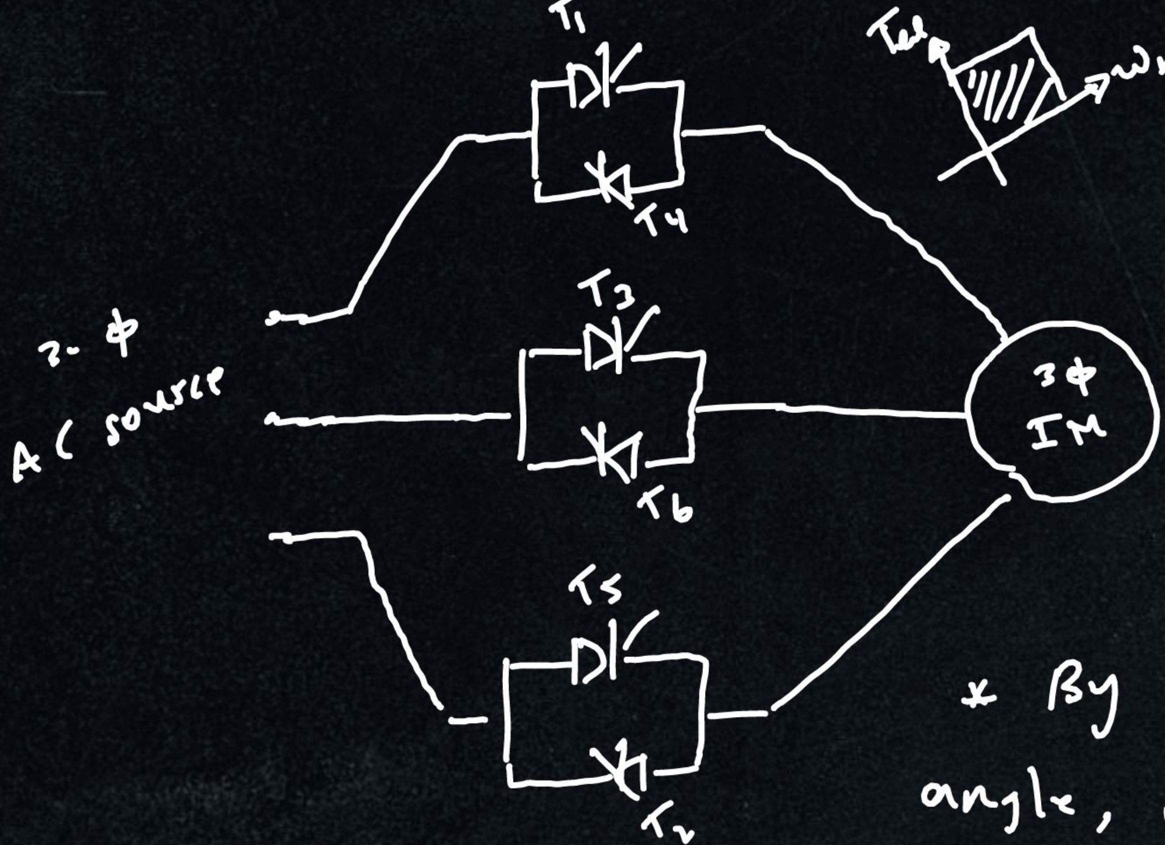
$V_3 < V_2 < V_1$

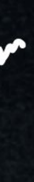
$V \downarrow \Rightarrow \omega_m \downarrow$

The variation of motor voltage is obtained by a 3-ϕ AC voltage regulator, which is able to change the motor voltage at fixed frequency.

- The change of voltage is obtained at the expense of a low PF and considerable amount of harmonics.
- The harmonics increase the losses and require derating of motor.
- The motor torque capability is also reduced.
- Applications: Fan + pump drives.

• structure of 3-Φ AC voltage regulator





* The gate signals must be synchronized with phase voltages and shifted from each other by 60°

* The machine operates only in the first quadrant

* By using an appropriate firing angle, the motor starting current (starting) torque is restricted by motor voltage reduction (soft starter)

⟶ increases the motor runtime.

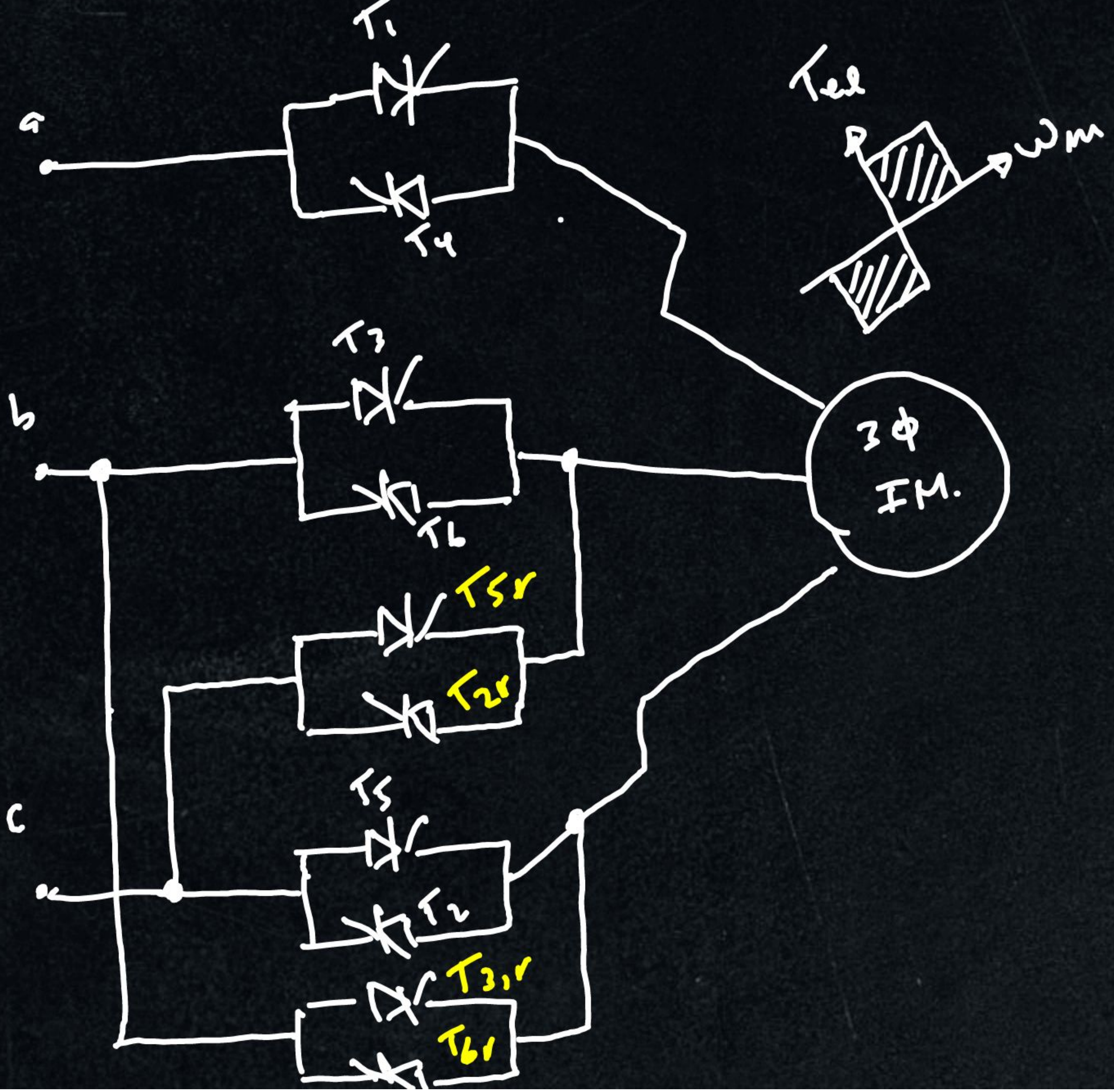
* The waveforms of motor voltages and currents are vary in three separate modes over the range of α.

* If α is constant, the motor voltage and current waveforms vary with the motor PF.

Therefore, the analysis of $3-\phi$ AC voltage regulator would be very complex because of interaction between the motor and controller.

* The controller output voltage (input of motor) depends on both the state of controller and the state of the motor $\rightarrow$ Simulation methods must be used.

Reversible Controller



- The sequence of gating for one direction (clockwise)

  $T_1\ T_2\ T_3\ T_4\ T_5\ T_6$

- The sequence of gating for counterclockwise rotation is

  $T_1\ T_{2r}\ T_{3r}\ T_4\ T_{5r}\ T_{6r}$

EX :- A pump has a torque-speed curve given by $T_L = C\omega_m^2$. It is proposed to use a 3ϕ IM. The pump speed is varying by using 3ϕ AC voltage regulator with speed reversible ability. The pump induction motor has 4 poles and Δ-connected. It has the following parameters referred to the stator:

$$R_s = 2.5\,\Omega,\ R_r' = 4.5\,\Omega,\ X_s = X_r' = 6\,\Omega,\ X_m = \text{very large}$$

The motor ratings are 6 kW, 400 V, 50 Hz, 1400 rpm

Calculate the RMS line voltage applied to the motor at speed of 1300 rpm.

$$T_{ed} = \frac{3V^2 (R_r'/s)}{\omega_s \left[\left((R_r'/s) + R_s \right)^2 + X_{eq}^2 \right]},$$

$$\omega_s = \frac{\omega_e}{P} = \frac{2\pi(50)}{2} = 50\pi \text{ r/sec}, \quad n_s = \frac{120}{P} f_e = \frac{120}{4}(50) = 1500 \text{ rpm}$$

$$R_r' = 4.5\,\Omega, \quad R_s = 2.5\,\Omega, \quad X_{eq} = 6 + 6 = 12\,\Omega$$

$$s = \frac{1500 - 1300}{1500} = \frac{2}{15}$$

$$T_{ed} = T_L = C\,\omega_m^2$$

$$T_{ed,r} = T_{L,r} = C\,\omega_{m,r}^2$$

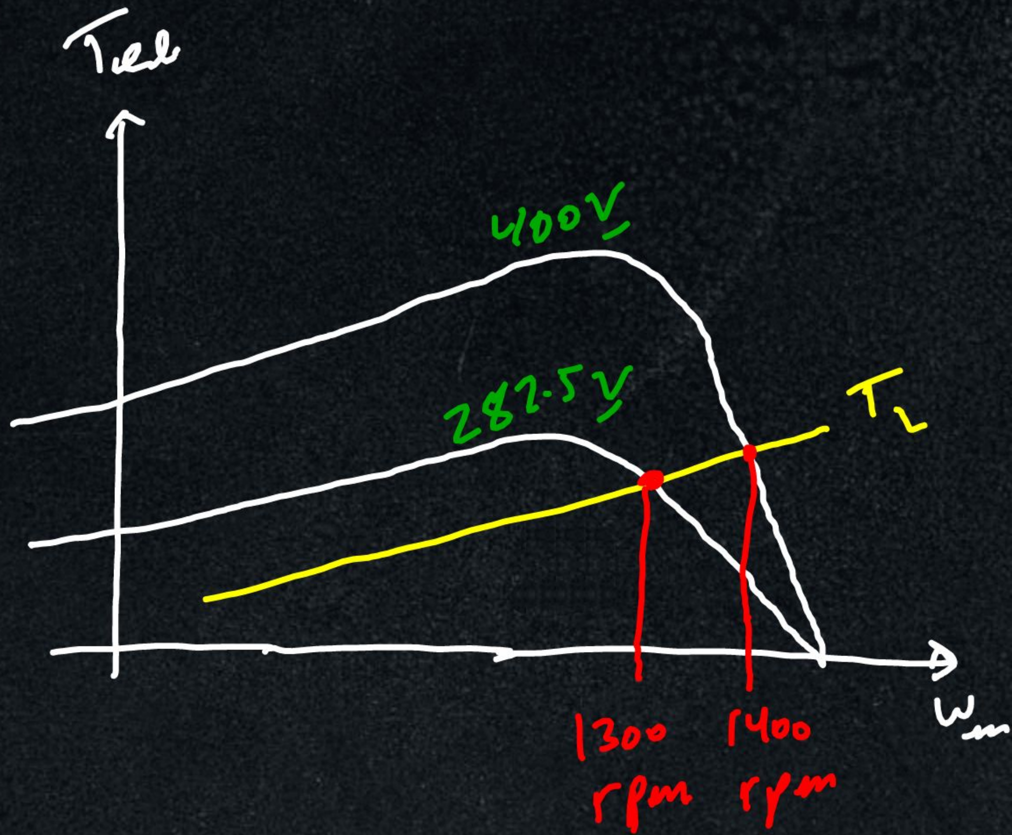
$$\frac{T_{ed}}{T_{ed,r}} = \left(\frac{\omega_m}{\omega_{m,r}} \right)^2 \Rightarrow T_{ed} = \left(\frac{1300}{1400} \right)^2 \frac{6000}{1400\frac{\pi}{30}}$$

$$T_{ed} = 35.28 \text{ N-m}$$

$$35.28 = \frac{3 V^2 \frac{4.5}{(2/15)}}{50\pi \left[\left(\frac{4.5}{(2/15)} + 2.5 \right)^2 + (12)^2 \right]}$$

solve for V

$$V = 282.5 \ \underline{V}$$



Fan and pump Drives

- In fan and pump drives, the load torque varies as the square of speed:

$$T_L \propto \omega_m^2 \Rightarrow T_L = C\omega_m^2 \quad ; \quad \omega_m = (1-\sigma)\omega_s$$

$$T_L = C(1-\sigma)^2\omega_s^2 \quad \dots (1)$$ where C is constant.

- The torque developed by the motor

$$T_{el} = \frac{3 I_r'^2 R_r'}{\omega_\sigma}$$

- Assume that the excitation current is neglected, then

$$T_{el} \approx \frac{3 I_s^2 R_r'}{\omega_\sigma} \quad \dots (2)$$

- If the friction and windage torques are neglected, then

$$T_{el} = T_L$$

$$\frac{3 I_s^2 R_r'}{\omega_r} = c(1-\sigma)^2 \omega_s^2 \quad ; \quad \omega_r = \sigma \omega_s$$

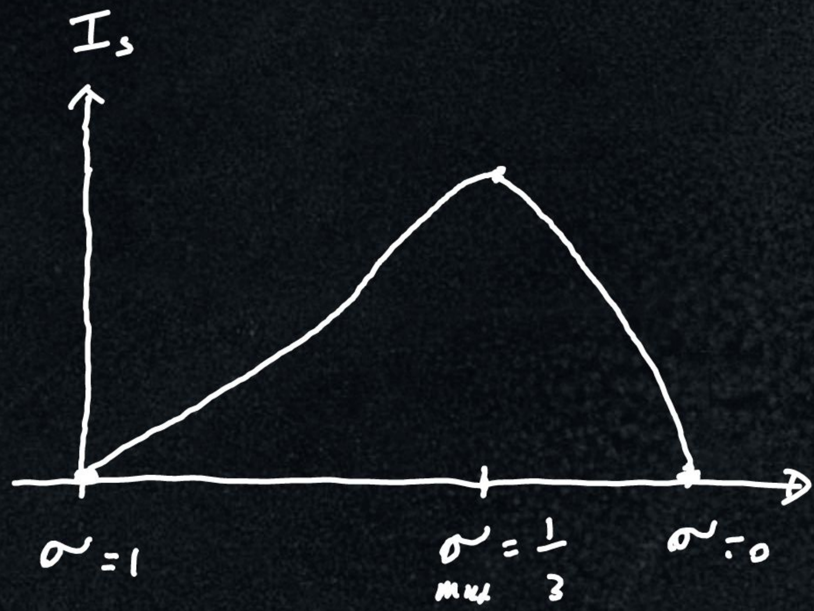
solve for I_s

$$I_s = K(1-\sigma)\sqrt{\sigma} \quad ; \quad K = \sqrt{\frac{c\,\omega_s^3}{3R_r'}}$$

- Maximum motor's current

$$I_s = K(1-\sigma)\sqrt{\sigma}$$

$$\frac{\partial I_s}{\partial \sigma} = 0 \Rightarrow \sigma_{max} = 1/3$$



$$I_{s,max} = \frac{2}{3\sqrt{3}} K$$

- Motor's rated current

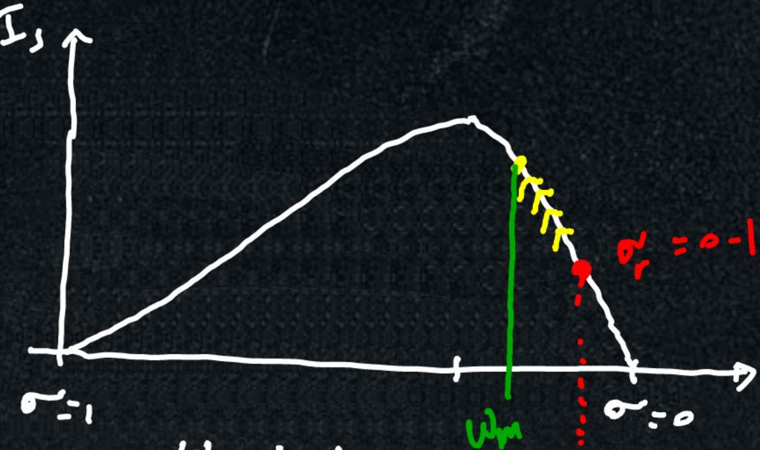
$$I_{s,r} = K(1-\sigma_r)\sqrt{\sigma_r}$$; σ_r : Rated slip.

Let $\gamma = \dfrac{I_{s,max}}{I_{s,r}}$; $I_{s,max} = \dfrac{2}{3\sqrt{3}} k$, $I_{s,r} = k(1-\sigma_r)\sqrt{\sigma_r}$

$$\gamma = \frac{2}{3\sqrt{3}(1-\sigma_r)\sqrt{\sigma_r}}$$

- If the motor rating power = full load power requirement

  $\Rightarrow$ the motor will be overloaded for speeds less than rated speed.



$\omega_m < \omega_{m,r}$

$\Rightarrow I_s > I_{s,r} \Rightarrow$ overloading

- Typical full-load slips in fan and pump drives $\sigma_r = 0.1 - 0.2$

$\gamma = 1.07 - 1.35 \Rightarrow$ Factor of overloading when a speed range from ω_r to $\frac{2}{3}\omega_r$ or to speed less than this is required, $I_{s,r}$ must be selected as equal to $I_{s,max}$.

$\sigma_r = 0.1 - 0.2$ ↘ $I_{s,r} = I_{s,max}$ ↗ $\sigma_{max} = 1/3$

⇒ Full load power delivered by the motor < Motor rating power

⇒ Motor is "derated"

Derating factor = DF = 0.74 - 0.93

- Additional derating should be applied for the reduction of cooling (shaft-mounted fan) at low speeds;
- Additional derating should be applied for additional heating due to harmonic losses.

② Variable Frequency Drive

- The relationship between synchronous speed and stator electric frequency is given by:-

$$\omega_s = \frac{\omega_e}{P} = \frac{2\pi f_e}{P}$$

- The speed of IM is very close to ω_s, and changing ω_s results in speed variations.

- Speed control is achieved in the inverter driven IM by means of variable frequency.

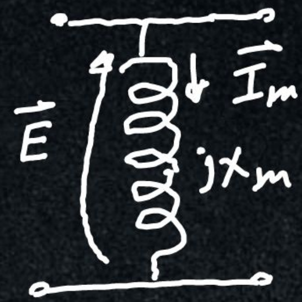
- Apart from frequency, the applied voltage needs to be varied to keep the air-gap flux, λ_m, constant and not let it saturated.

- The rms value of air-gap flux is

$$\lambda_m = L_m I_m$$

$I_m \rightarrow$ magnetizing current

$L_m \rightarrow$ magnetizing inductance.



$$I_m = \frac{E}{X_m} = \frac{E}{\omega_e L_m}$$

$$\Rightarrow \lambda_m = \cancel{L_m} \frac{E}{\omega_e \cancel{L_m}} \Rightarrow \lambda_m = \frac{E}{\omega_e} \approx \frac{V_s}{\omega_e}$$

$V_s \rightarrow$ stator rms phase voltage

A number of control strategies have been formulated, depending on how the voltage-to-frequency ratio is implemented:

(i) Constant Volts/Hz control or VVVF

Variable Voltage
Variable Frequency.

(ii) Constant air-gap flux control
(scalar control)

(iii) Vector control &

(i) Constant Volts / Hz control (VVVF drive)

$$\lambda_m \approx \frac{V_s}{\omega_e}$$

To maintain λ_m constant, $\frac{V_s}{\omega_e}$ has to be maintained constant $\rightarrow$ whenever ω_e is changed to control the speed, V_s has to be changed accordingly to maintain λ_m constant.

-

In general, the relationship between V_s and ω_e is written as:

$$V_s = K_{vf}\,\omega_e + V_0 \; ; \; K_{vf} = \frac{V_{s,r}}{\omega_{e,r}}$$

10 V

220

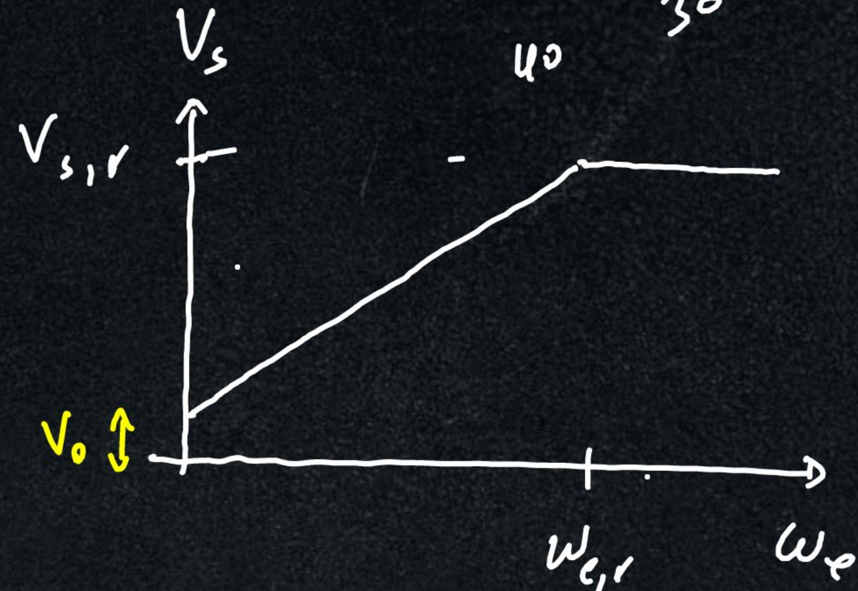
110

80

40

30

where V_0 is the offset voltage to overcome the voltage drop across R_s at low speeds



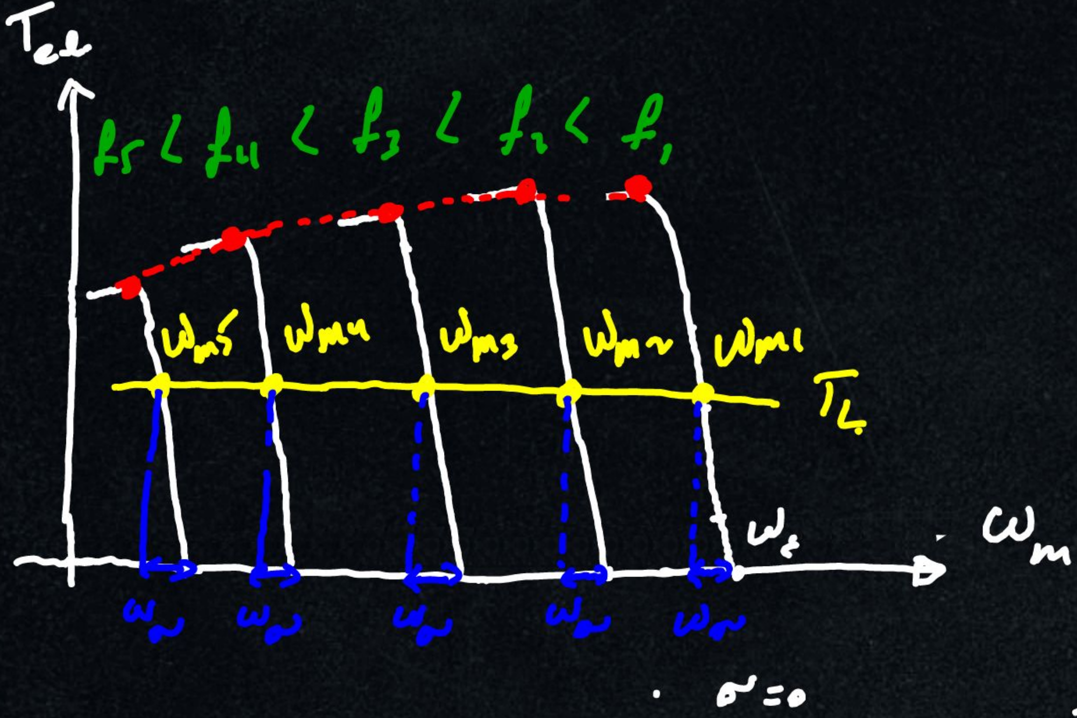
$$T_{el} = \frac{3 V_s^2 (R_r'/\sigma)}{\omega_s \left[\left((R_r'/\sigma) + R_s \right)^2 + X_{eq}^2 \right]}$$

$$T_{el,max} = \frac{3 V_s^2}{2 \omega_s \left[R_s + \sqrt{R_s^2 + X_{eq}^2} \right]}$$

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_s^2 + X_{eq}^2}}$$

since σ is small $\Rightarrow \frac{R_r'}{\sigma}$ is large

$$T_{el} \approx \frac{3 V_s^2}{\omega_s R_r'} \sigma = \frac{3}{R_r'} \frac{V_s^2}{\omega_s^2} \sigma \omega_s = K \omega_\sigma$$



$f \downarrow \Rightarrow \omega_e \downarrow \Rightarrow \omega_r \downarrow$

$\Rightarrow \omega_m \downarrow.$

$T_{el} = K\,\omega_\sigma$

If $T_{el} = T_r = T_L = \text{constant}$

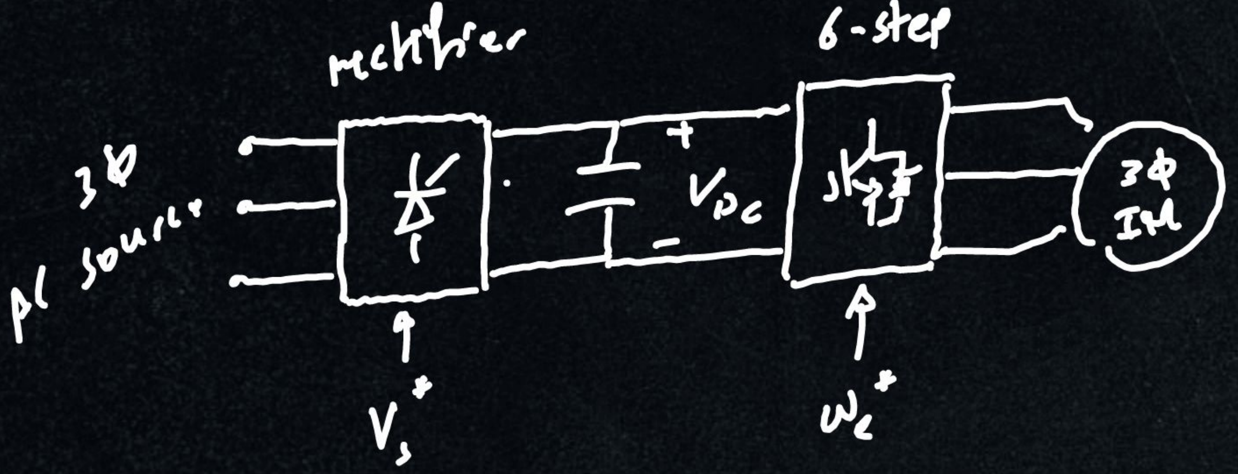
$\Rightarrow \omega_\sigma$ is constant

$\omega_\sigma = \omega_r - \omega_m$

$\omega_\sigma = \omega_{r_1} - \omega_{m_1} = \omega_{r_2} - \omega_{m_2}$ if T_{el} is constant.

Drive Circuits

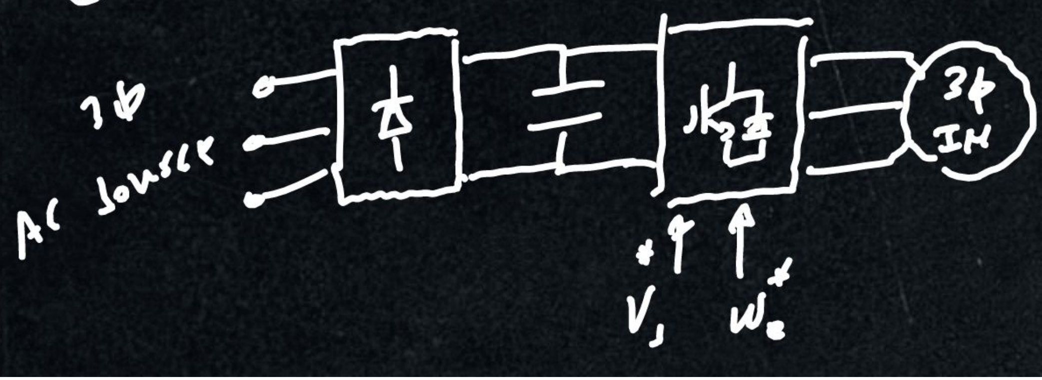
① 6-step inverter with controlled rectifier

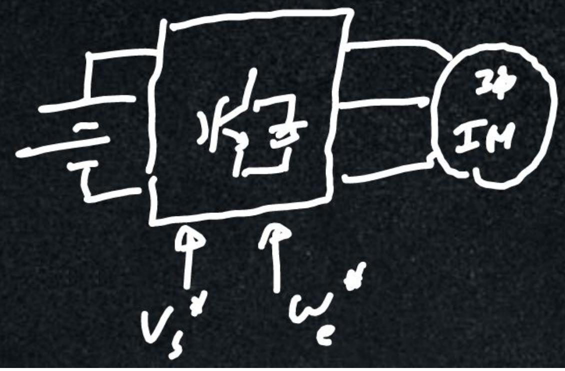


$$V_{1,peak} = \frac{2}{\pi} V_{DC}$$

$$V_s = \frac{2}{\sqrt{2}\,\pi} V_{DC} = \frac{\sqrt{2}}{\pi} V_{DC}$$

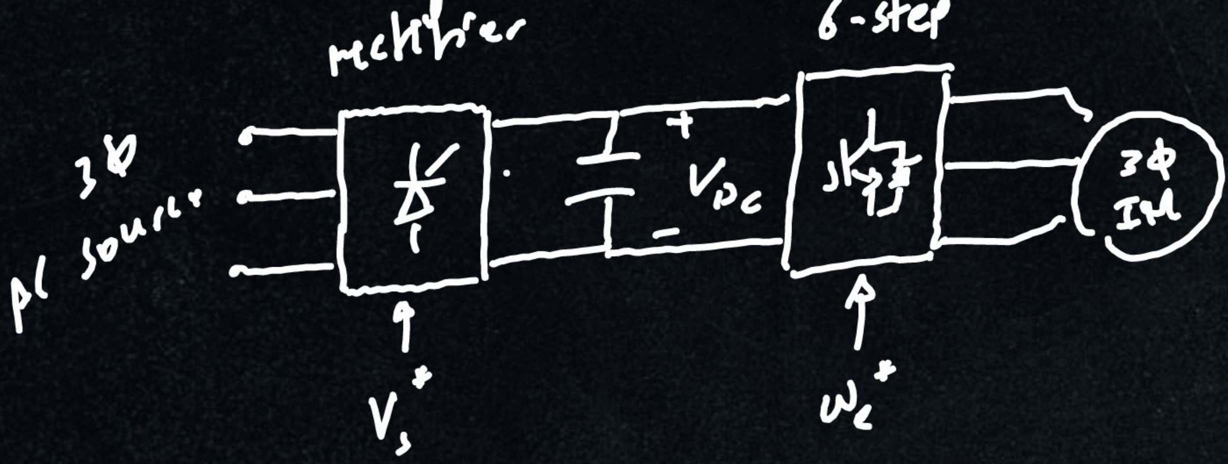
② Three-phase ~~inverter~~ inverter





Drive Circuits

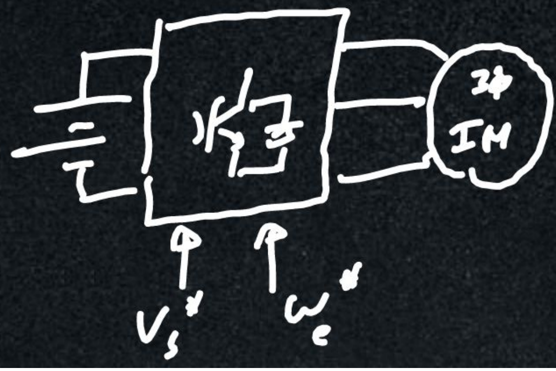
① 6-step inverter with controlled rectifier



$$V_{1,peak} = \frac{2}{\pi} V_{DC}$$

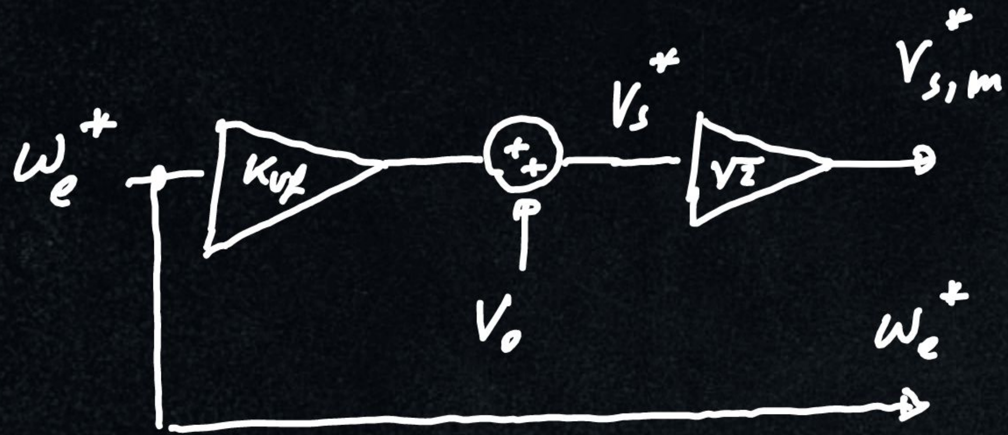
$$V_s = \frac{2}{\sqrt{2}\,\pi} V_{DC} = \frac{\sqrt{2}}{\pi} V_{DC}$$

② Three-phase inverter inverter



Control system

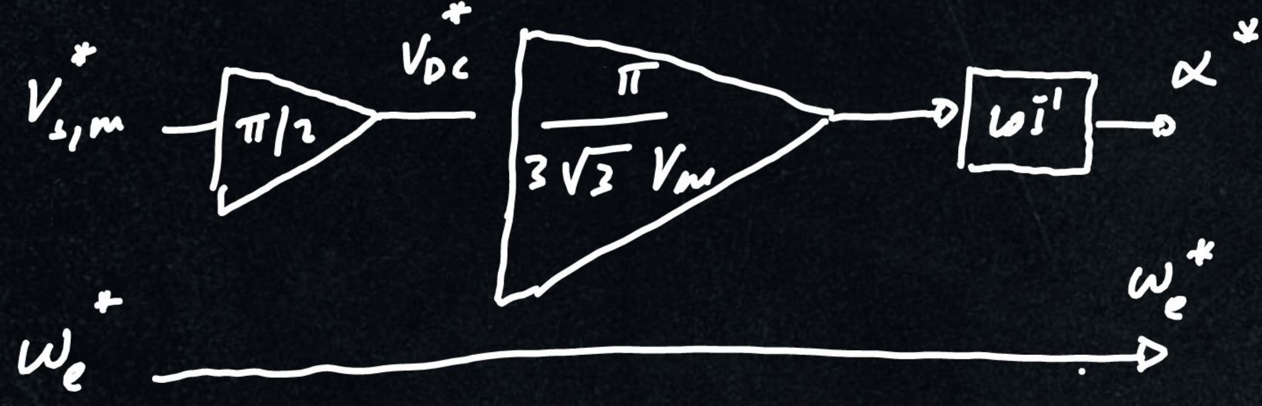
1) Open loop



$V_s^* = K_{vf}\,\omega_e^* + V_0 \;;\; K_{vf} = \dfrac{V_{s,r}}{\omega_{e,r}}$

$V_{s,m}$ = (peak) stator phase voltage

6-step inverter with controlled rectifier "Full-wave"



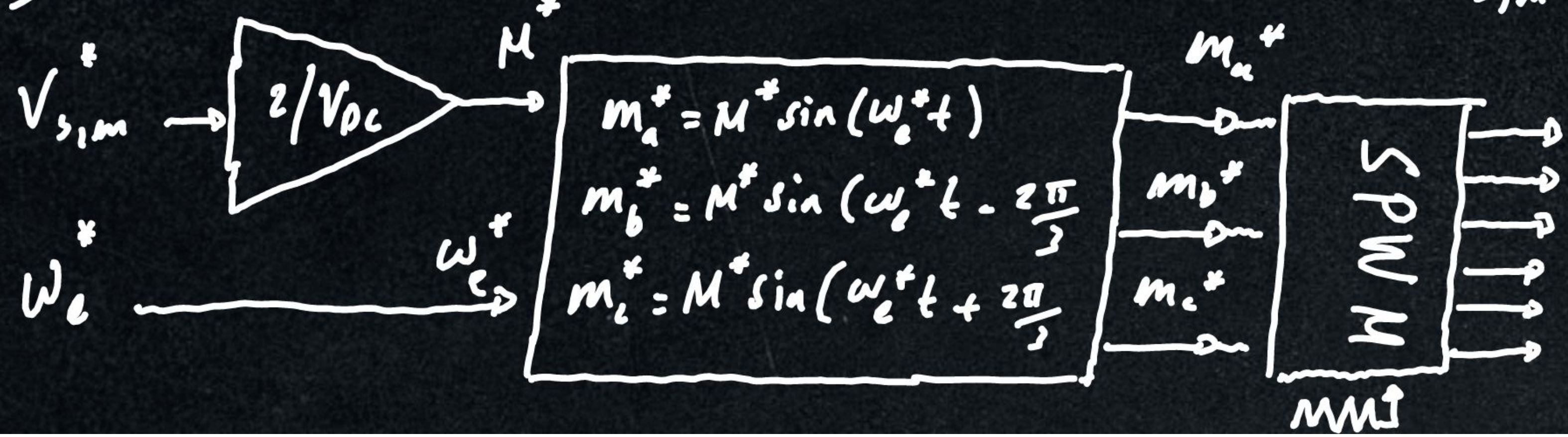
$$V_{s,m} = \frac{2}{\pi} V_{DC}$$

$$V_{DC} = \frac{3\sqrt{3}\,V_m}{\pi} \cos\alpha$$

where V_m is the phase voltage of AC source (peak)

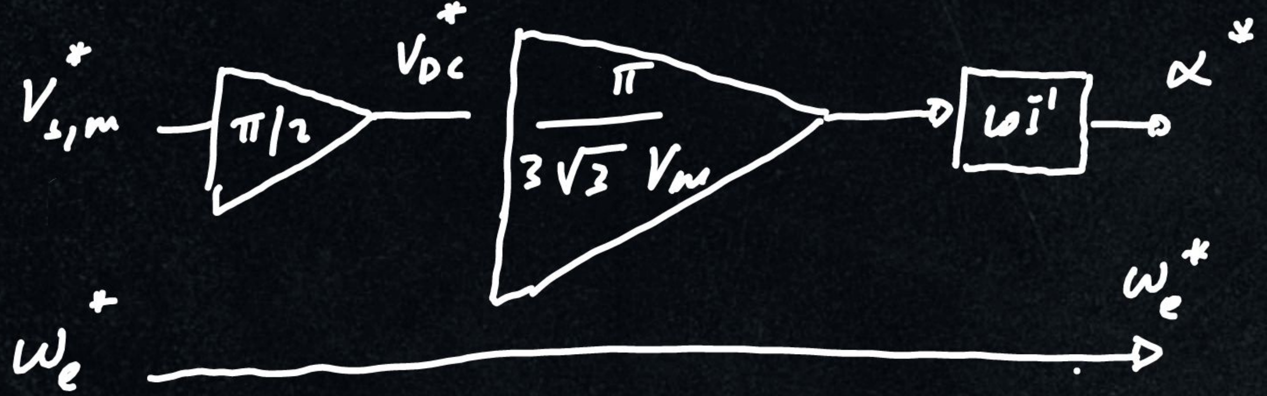
3ϕ inverter

$$V_{s,m}^* = M^* \frac{V_{DC}}{2}$$



gate signals

6-step inverter with controlled rectifier "Full-wave"



$$V_{s,m} = \frac{2}{\pi} V_{DC}$$

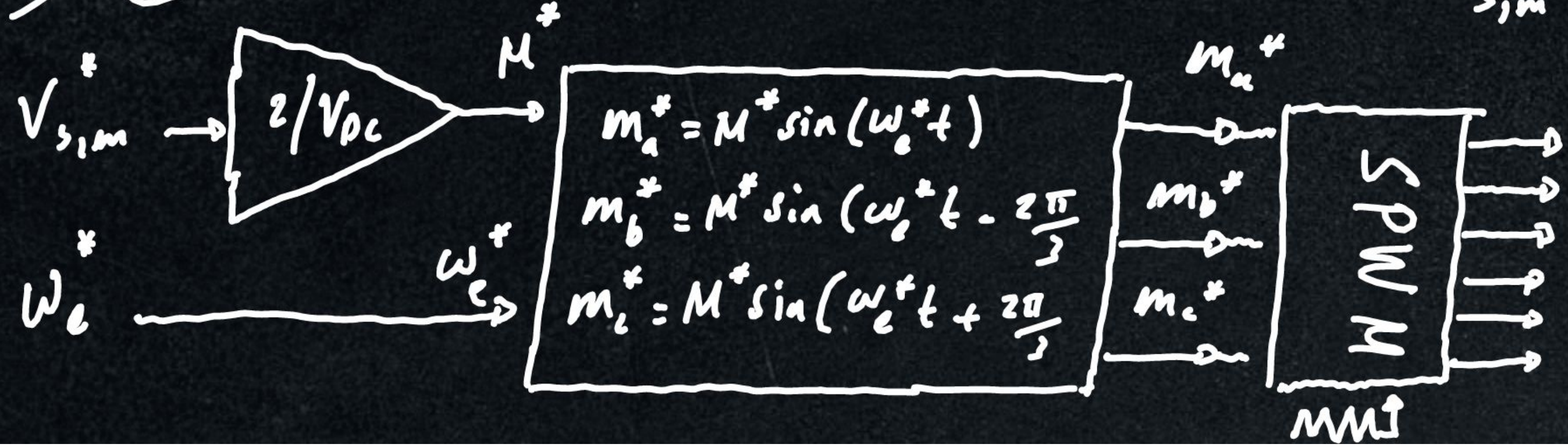
$$V_{DC} = \frac{3\sqrt{3}\,V_m}{\pi} \cos\alpha$$

where V_m is the phase voltage of AC source (peak)

$\omega_e \Uparrow \wedge$

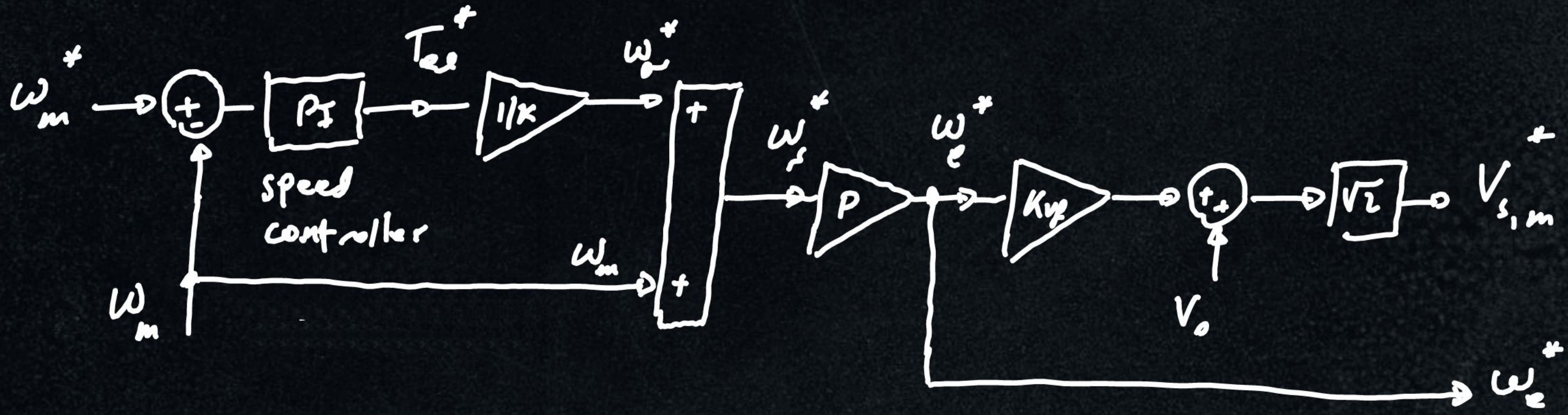
ω_m

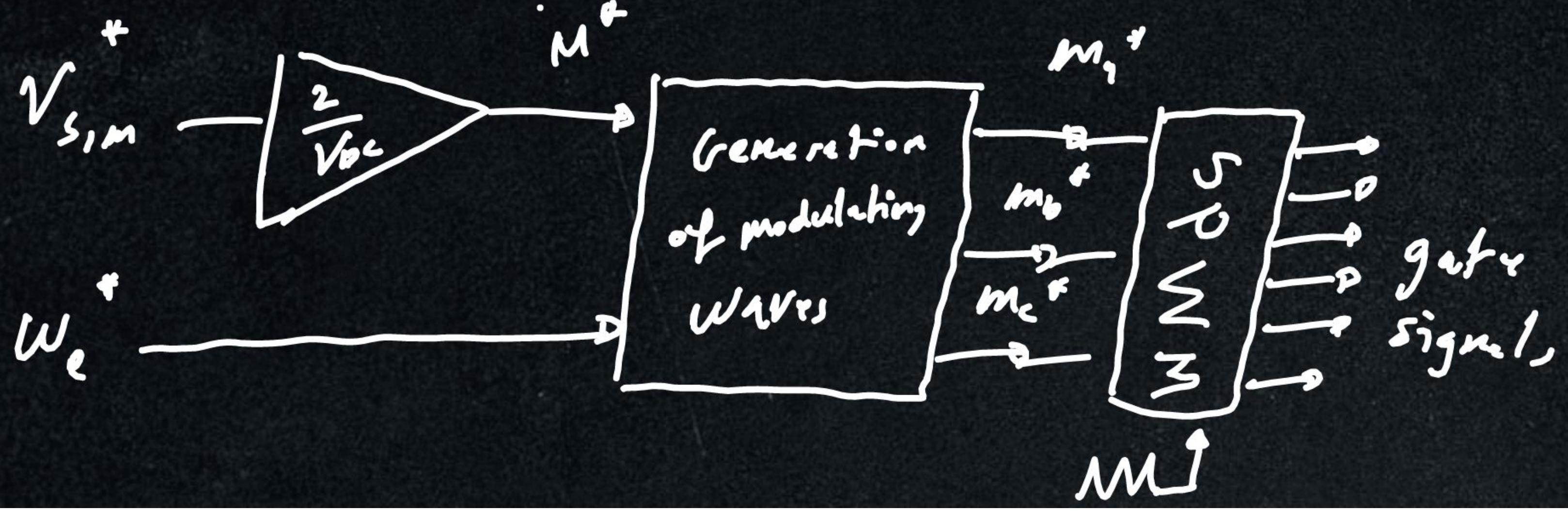
3 ϕ inverter



$$V_{s,m}^* = M^* \frac{V_{DC}}{2}$$

Closed loop (with 3φ inverter)

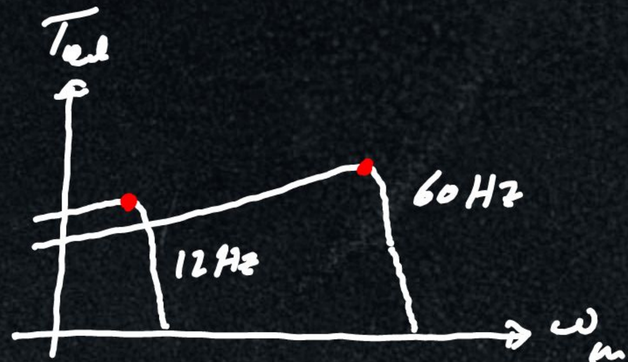




EX :- A 3ϕ Y-connected, 60 Hz, 4 poles IM has the following parameters: $R_s = R_r' = 0.024\ \Omega$, $X_s = X_r' = 0.12\ \Omega$

The motor is controlled using VVVF drive. For an operating frequency of 12 Hz, Calculate

i) The maximum torque as a ratio of its value at the rated frequency.



$$\frac{T_{el,max}(12\,Hz)}{T_{el,max}(60\,Hz)} \quad ??$$

$$\frac{V_s}{f_e} = \text{constant} = \frac{V_s}{\omega_s}$$

$$T_{el,max} = \frac{3V_s^2}{2\omega_s^2\left[R_s + \sqrt{R_s^2 + X_{eq}^2}\right]}\,\frac{1}{P}\,2\pi(f_e)$$

$$\frac{T_{el,max}(12)}{T_{el,max}(60)} = \frac{R_s + \sqrt{R_s^2 + (2\pi \times 60 \times L_{eq})^2}}{R_s + \sqrt{R_s^2 + (2\pi \times 12 \times L_{eq})^2}} \cdot \frac{12}{60} = \boxed{0.68}$$

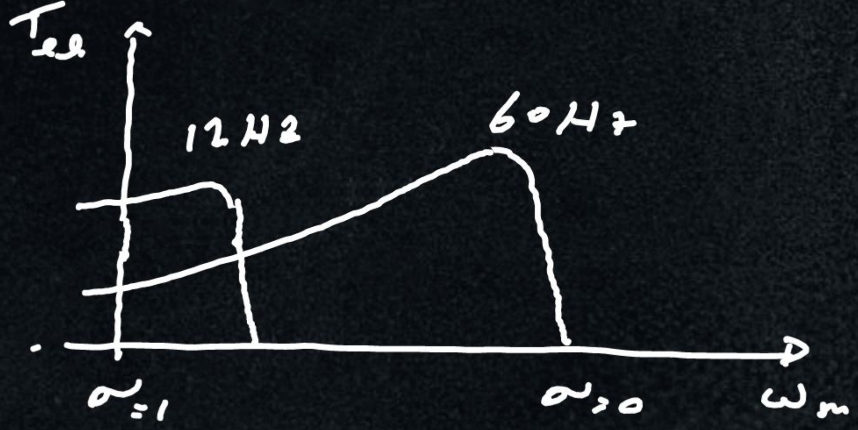
$$X_{eq} = X_s + X_r' = 0.12 + 0.12 \quad @ \ 60\,Hz \quad , \quad R_s = 0.024\,\Omega$$

$$L_{eq} = \frac{0.24}{2\pi \times 60}$$

2) The starting torque as a ratio of its value at the rated frequency.

$$T_{el} = \frac{3V_s^2}{\omega_s \left[\left(R_s + \frac{R_r'}{\sigma} \right)^2 + X_{eq}^2 \right]} \cdot \frac{R_r'}{\sigma} \quad ; \quad T_{el,start} \Big|_{\sigma = 1}$$

$$\frac{T_{el,start}(12)}{T_{el,start}(60)} = \gamma$$



$$T_{el,start} = \frac{3V_s^2}{\omega_s^2\left[(R_s+R_r')^2 + X_{eq}^2\right]} R_r' \frac{1}{P}(2\pi) f_e$$

$$\gamma = \frac{(R_s+R_r')^2 + (2\pi \times 60 \times L_{eq})^2}{(R_s+R_r')^2 + (2\pi \times 12 \times L_{eq})^2} \cdot \frac{12}{60} = 2.6$$

(ii) Constant air-gap flux control

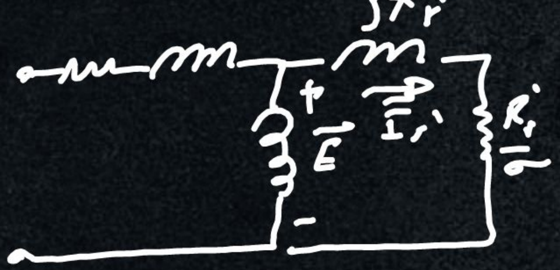
* It resolves IM into an equivalent separately excited DC motor in terms of its speed response, but not in terms of decoupling of the flux and torque channels.

* Recall the air-gap flux linkage equation :-

$$\lambda_m = L_m I_m = L_m \frac{E}{\omega_e L_m} = \frac{E}{\omega_e}$$

$$T_{el} = \frac{3 I_r'^2 R_r'}{\omega_\sigma}$$

$$\lambda_m = \frac{E}{\omega_e} \quad , \quad T_{el} = \frac{3 I_r'^2 R_r'}{\omega_\sigma}$$



$$\vec{I_r'} = \frac{\vec{E}}{\frac{R_r'}{\sigma} + jX_r'} \Rightarrow I_r' = \frac{E}{\sqrt{\left(\frac{R_r'}{\sigma}\right)^2 + X_r'^2}}$$

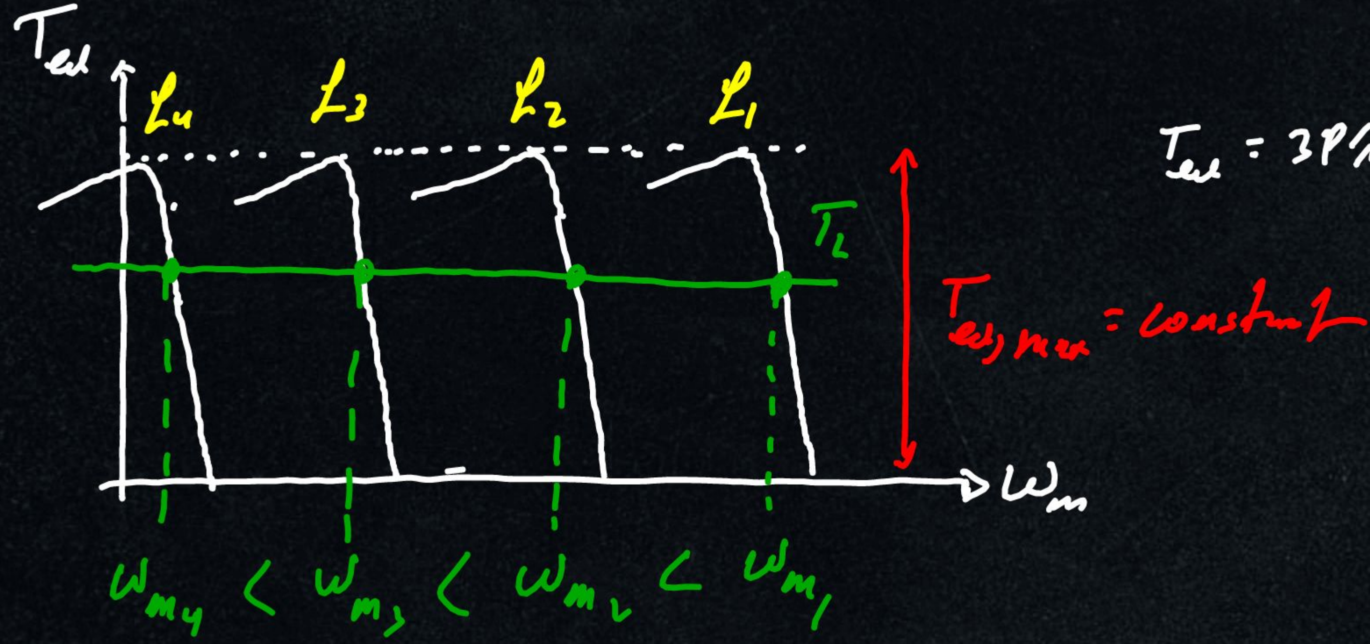
$$T_{el} = \frac{3E^2 R_r'}{\omega_\sigma\left[\left(\frac{R_r'}{\sigma}\right)^2 + X_r'^2\right]} = \frac{3E^2 R_r'}{\omega_e^2 \omega_\sigma \left[\left(\frac{R_r'}{\sigma\omega_e}\right)^2 + L_r'^2\right]}$$

$$\omega_\sigma = \sigma\omega_s$$

$$= \frac{\sigma\omega_e}{p}$$

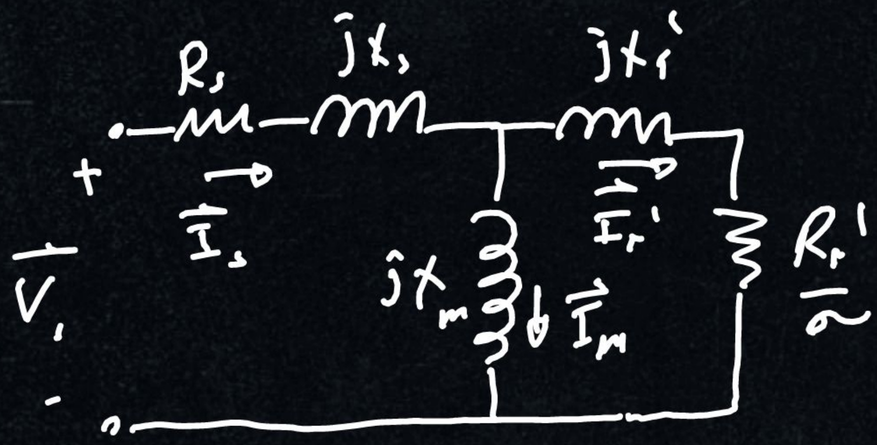
$$\sigma\omega_e = p\,\omega_\sigma$$

$$T_{el} = \frac{3\lambda_m^2 R_r'}{\omega_\sigma} \frac{1}{\left(\frac{R_r'}{p\omega_\sigma}\right)^2 + L_r'^2} \Rightarrow \boxed{T_{el} = 3p\lambda_m^2 \frac{R_r'/(p\omega_\sigma)}{\left(R_r'/(p\omega_\sigma)\right)^2 + L_r'^2}}$$



$$T_{el} = 3p\lambda_m^2 \frac{R_r'/(p\omega_\sigma)}{\left(R_r'/(p\omega_\sigma)\right)^2 + L_r''}$$

stator current control

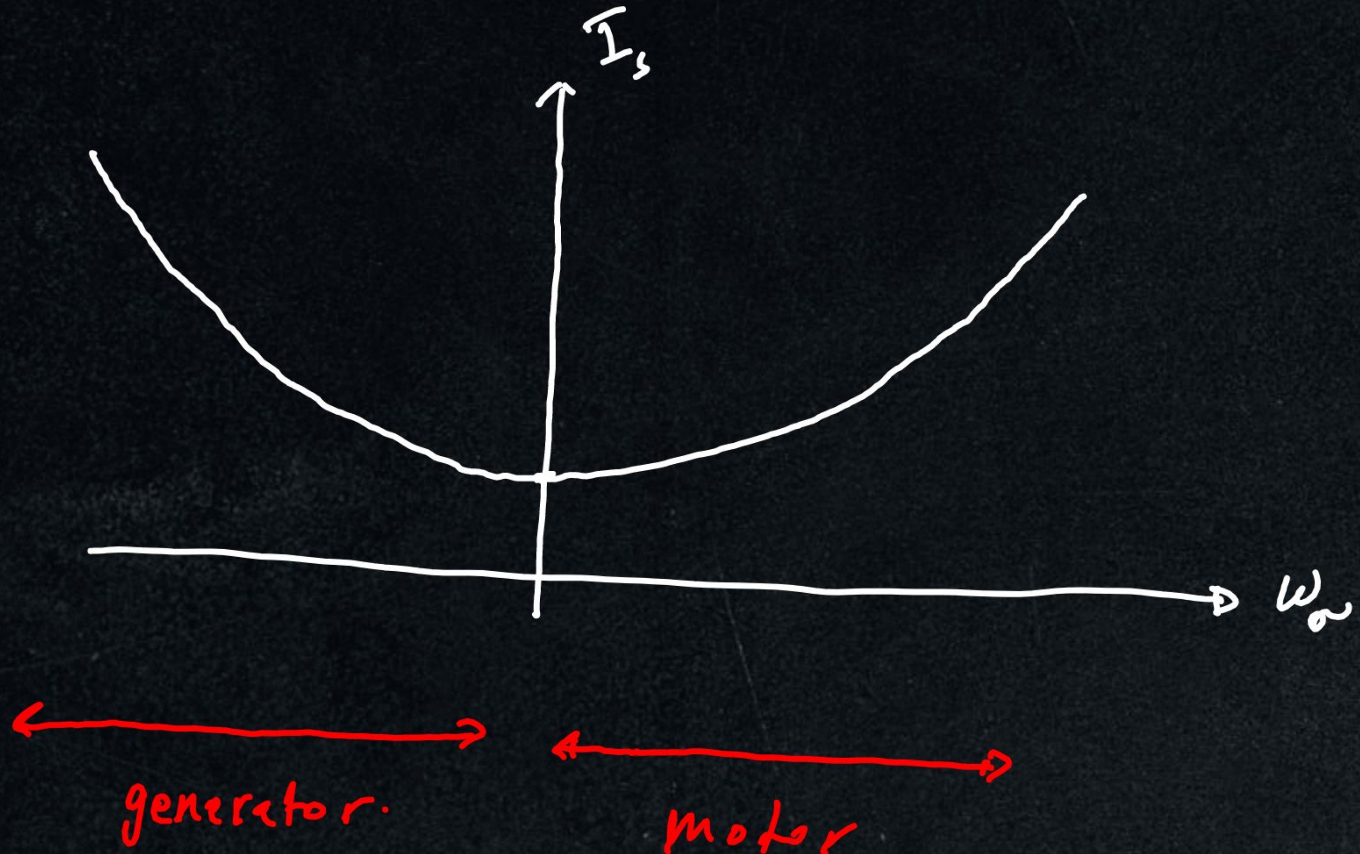


$$\vec{I}_m' = \frac{(R_r'/\sigma) + jX_r'}{(R_r'/\sigma) + j(X_r' + X_m)} \cdot \vec{I}_s$$

$$I_m = \sqrt{\frac{(R_r'/\sigma)^2 + (X_r')^2}{(R_r'/\sigma)^2 + (X_r' + X_m)^2}} \cdot I_s = \sqrt{\frac{(R_r'/(\sigma\omega_e))^2 + L_r'^2}{(R_r'/(\sigma\omega_e))^2 + (L_r' + L_m)^2}} \cdot I_s$$

$$\sigma\omega_e = P\omega_\sigma \;;\; \omega_\sigma = \sigma\omega_s = \frac{\sigma\omega_e}{P}$$

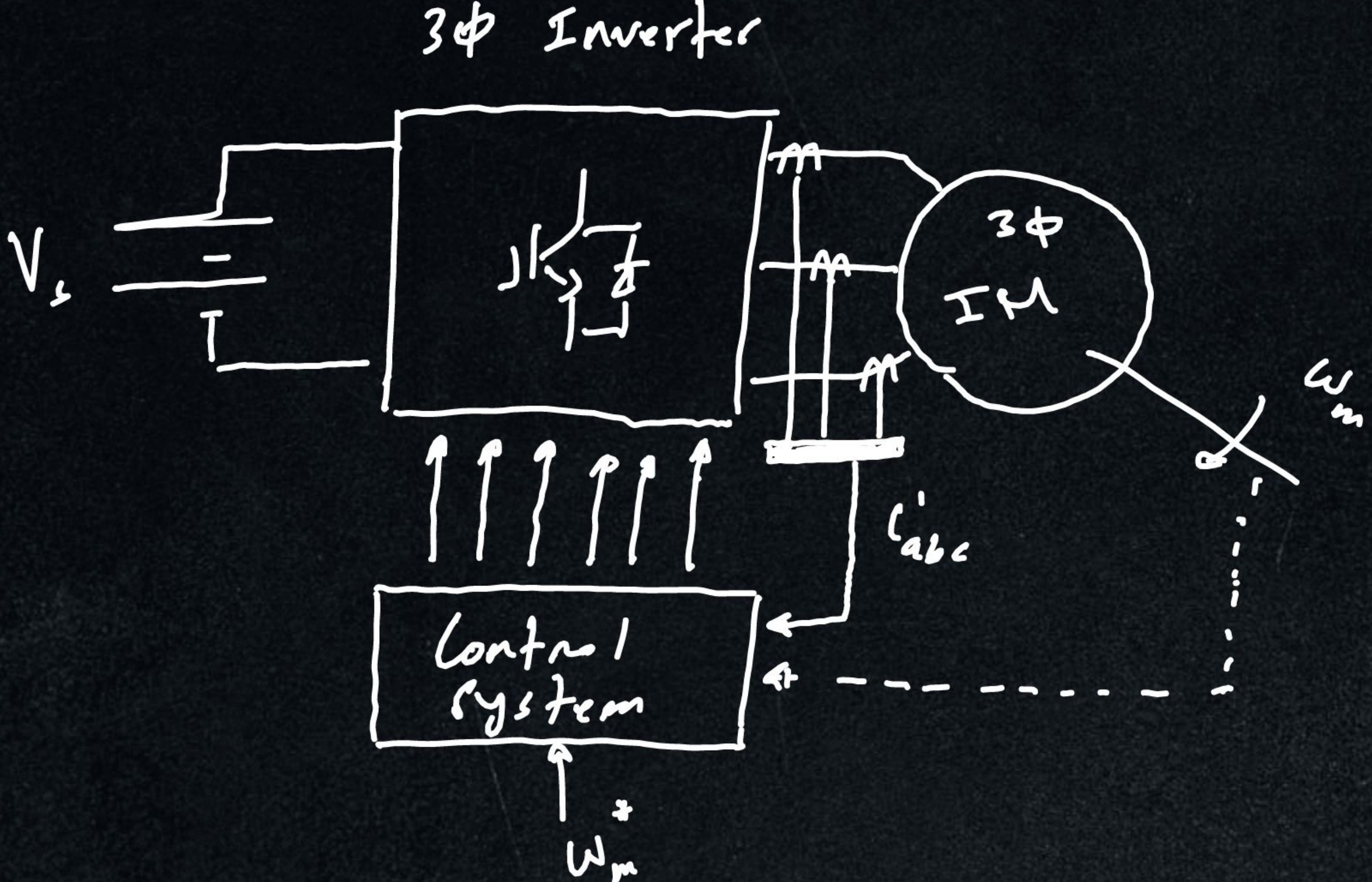
1. $$I_s = I_m \sqrt{\frac{\left(R_r'/(p\omega_{r})\right)^2 + (L_r' + L_m)^2}{\left(R_r'/(p\omega_{r})\right)^2 + L_r'^2}}.$$



$T_{el} = F_1(\omega_r)$

$I_s = F_2(\omega_r)$

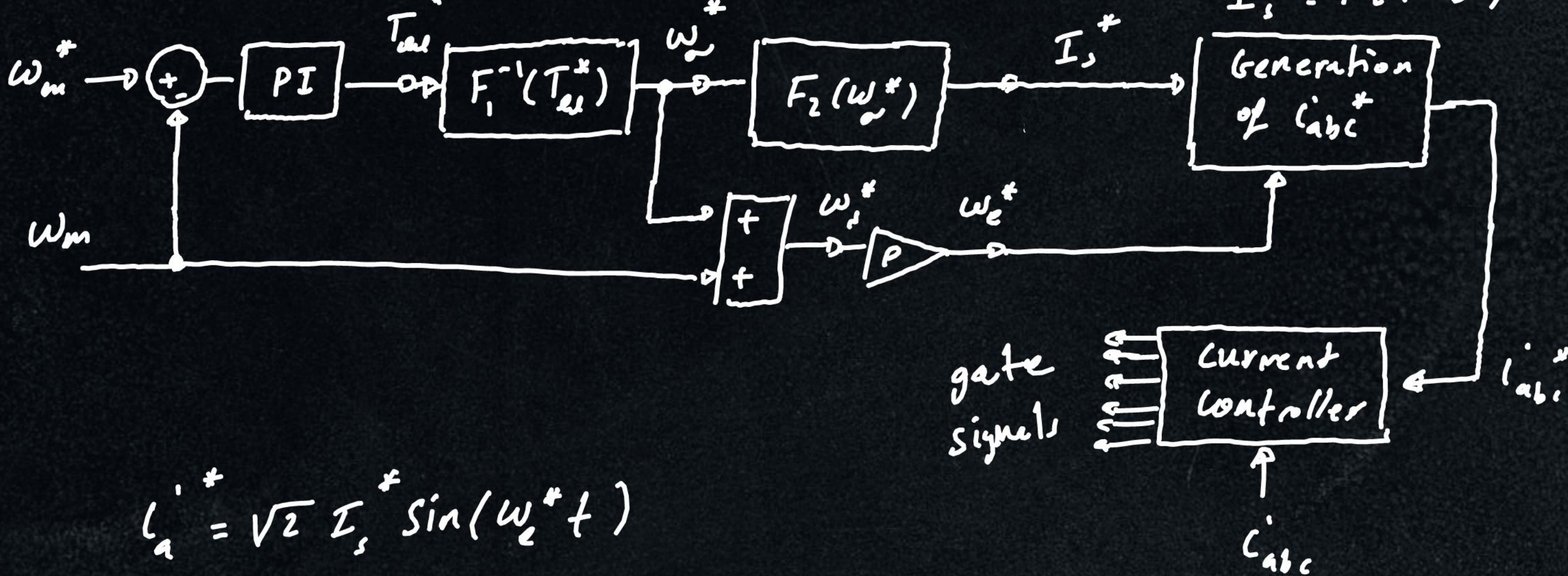
Drive Circuit



Control system

$T_{el} = F_1(\omega_{sl})$

$I_s = F_2(\omega_{sl})$



$$i_a^* = \sqrt{2}\, I_s^* \sin(\omega_e^* t)$$

$$i_b^* = \sqrt{2}\, I_s^* \sin(\omega_e^* t - 2\pi/3)$$

$$i_c^* = \sqrt{2}\, I_s^* \sin(\omega_e^* t + 2\pi/3)$$

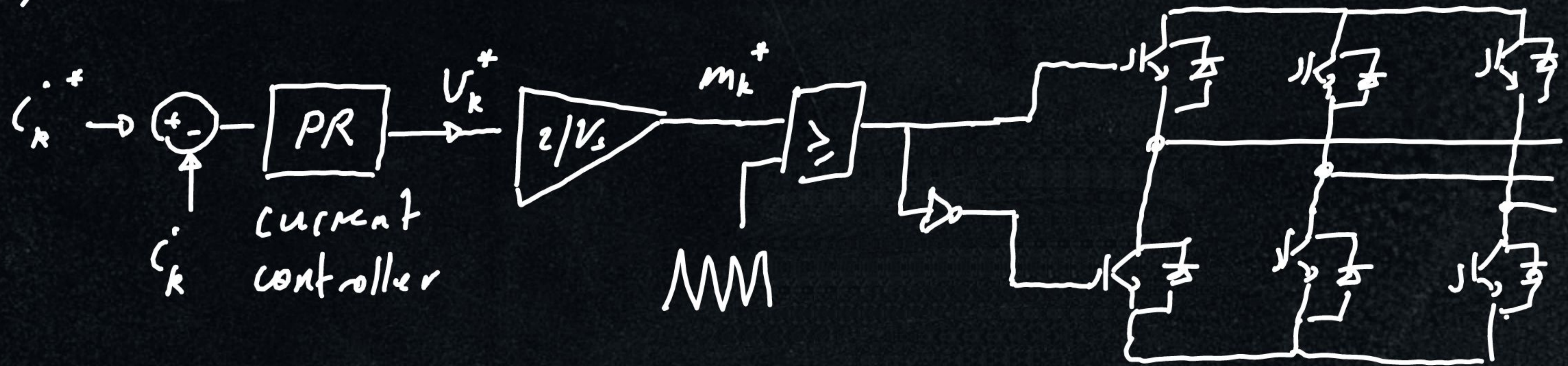
current controller

- SPWM current control
- HCC.

Current Controller

$$\frac{V_{pm}}{} = \mu \frac{V_{DC}}{2}$$

1) SPWM current control $k \in \{a, b, c\}$

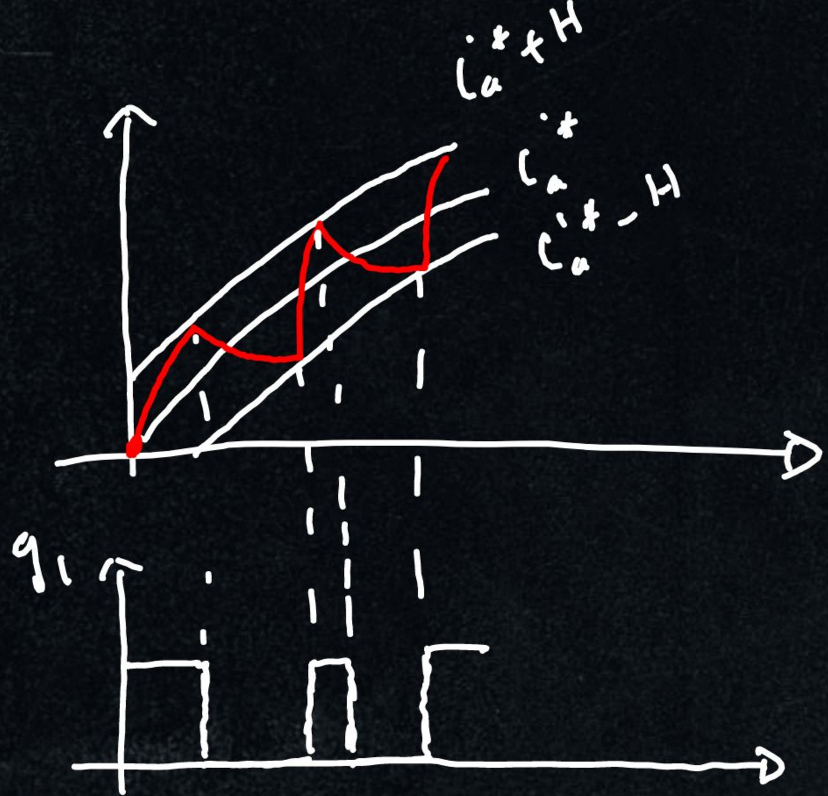


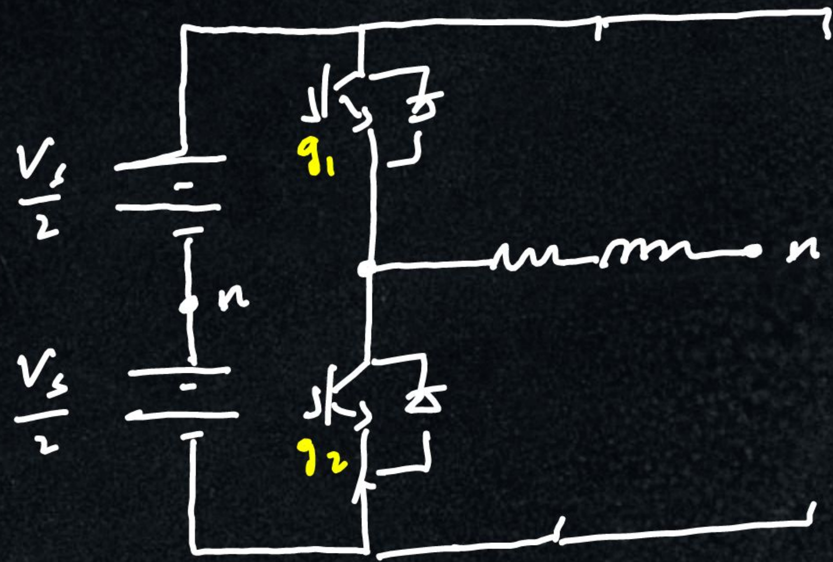
PR: proportional Resonant

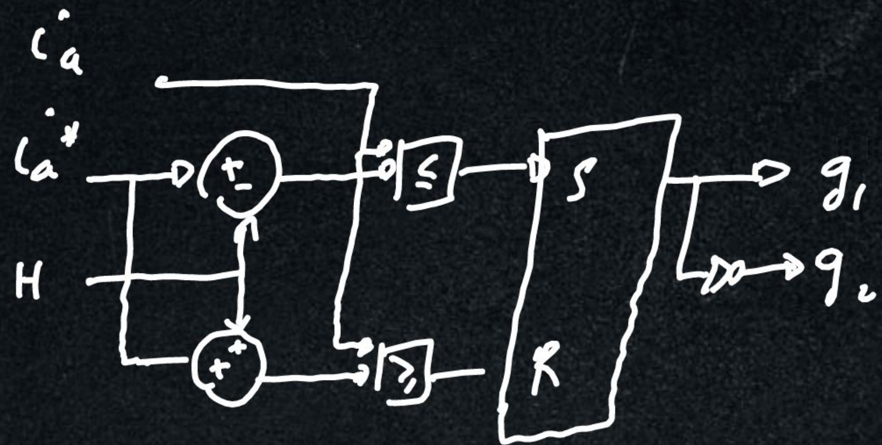
$$PI = k_1 + \frac{k_2}{s} \qquad s = j\omega$$

$$PR = k_1 + \frac{k_r s}{s^2 + \omega_e^2} \qquad \text{Error} \propto \frac{1}{PI}$$

2) HCC







EX :- A 400V, 50 Hz, 6 pole, 960 rpm, Y-connected IM. The parameters per-phase referred to the stator:

$R_s = 0.4\ \Omega,\ R_r' = 0.2\ \Omega,\ X_s = X_r' = 1.5\ \Omega,\ X_m = 30\ \Omega$

The motor is controlled using constant air-gap flux controllers.

1) Calculate the rated slip, slip speed, and induced voltage

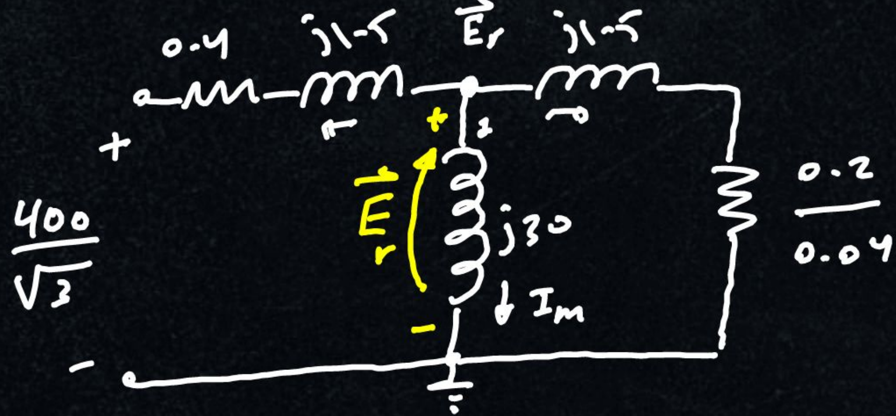
$\sigma_r,\ \omega_{\sigma,r},\ E_r$

$$\omega_{f,r} = \frac{\omega_{e,r}}{p} = \frac{2\pi \times 50}{3} = 104.72 \text{ r/sec}$$

$$\omega_{m,r} = 960\ \frac{\pi}{30} = 100.53 \text{ r/sec}$$

$$\sigma_r = \frac{\omega_{f,r} - \omega_{m,r}}{\omega_{f,r}} = \frac{104.72 - 100.53}{104.72} = 0.04$$

$$\omega_{\sigma,r} = \sigma_r\, \omega_{f,r} = 0.04(104.72) = 4.2 \text{ r/sec}$$



$$\frac{\vec{E}_r - \frac{400}{\sqrt{3}}}{0.4 + j1.5} + \frac{\vec{E}_r}{j30} + \frac{\vec{E}_r}{\frac{0.2}{0.04} + j1.5} = 0$$

solve for $\vec{E}_r$

$E_r = 189\ \underline{V}$

2) Calculate the air-gap flux

$$\lambda_{m,r} = L_m I_{m,r} = \cancel{L_m} \frac{E_r}{\omega_{e,r} \cancel{L_m}} = \frac{E_r}{\omega_{e,r}} = \frac{189}{2\pi \times 50} = 0.63\ Wb$$

or H·A.

3) Calculate the rated torque developed by the motor

$$T_{el,r} = 3P\lambda_{m,r}^2 \frac{R_r'/(P\omega_{\sigma,r})}{(R_r'/(P\omega_{\sigma,r}))^2 + L_r'^2}$$

$P = 3$

$\lambda_{m,r} = 0.63$

$R_r' = 0.2$

$\omega_{\sigma,r} = 4.2$

$L_r' = \frac{1.5}{2\pi \times 50}$

$\boxed{T_{el,r} = 188\ N \cdot m}$

4) Calculate the motor speed and at half the rated torque and 25 Hz.

$$T_{el} = 3P\lambda_m^2 \frac{R_r'/(P\omega_{sl})}{(R_r'/(P\omega_{sl}))^2 + L_s'^2}$$

$$\frac{188}{2} = 3(3)(0.63)^2 \frac{(0.2/(3\omega_{sl}))}{(0.2/(3\omega_{sl}))^2 + \left(\frac{1.5}{2\pi \times 50}\right)^2}$$

solve for $\omega_{sl} \Rightarrow \omega_{sl} = 1.96$ rad/sec

$$\omega_m = \omega_s - \omega_{sl} = \frac{2\pi \times 25}{3} - 1.96 \Rightarrow \boxed{\omega_m = 50.4 \text{ r/sec}}$$

$$I_s = I_m \sqrt{\frac{\left(R_r'/(p\omega_{\sigma})\right)^2 + \left(L_{\sigma r}' + L_m\right)^2}{\left(R_r'/(p\omega_{\sigma})\right)^2 + L_r'^2}}$$

$$\lambda_m = L_m I_m$$

$$\lambda_m = 0.63$$

$$L_m = \frac{30}{2\pi \times 50}$$

$$I_s = \frac{\lambda_m}{L_m} \sqrt{\frac{\left(0.2/(3(1.96))\right)^2 + \left(\frac{31.5}{2\pi \times 50}\right)^2}{\left(0.2/(3(1.96))\right)^2 + \left(\frac{1.5}{2\pi \times 50}\right)^2}}$$

$$\boxed{I_s = 19.85\ A}$$

Field Weakening

$$\lambda_m = L_m I_m = L_m \frac{E}{X_m}$$

$$\lambda_m = \frac{E}{\omega_e} \approx \frac{V_s}{\omega_e}$$

when $\omega_m \leq \omega_{m,r} \Rightarrow \lambda_m = \lambda_{m,r}$

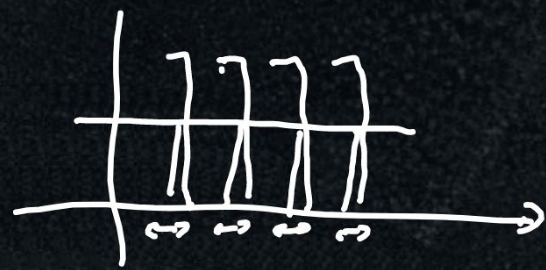
when $\omega_m \geq \omega_{m,r} \Rightarrow V_s = V_{s,r} = \omega_e \lambda_m = \omega_{e,r} \lambda_{m,r}$

$$\Rightarrow \lambda_m = \lambda_{m,r} \frac{\omega_{e,r}}{\omega_e}$$ "Field weakening"

① $T_{el,max}$

$$T_{el,max} = \frac{3P V_{s,r}^2}{2\omega_e \left[R_s + \sqrt{R_s^2 + X_{eq}^2} \right]} \approx \frac{3p V_{s,r}^2}{2\omega_e^2 L_{eq}}$$

$$T_{el,max} \propto \frac{1}{\omega_e^2}$$



② ω_σ

$$I_s \approx \frac{V_{s,r}}{\sqrt{\left(\frac{R_r'}{\sigma} + R_s\right)^2 + X_{eq}^2}}$$

since σ is very small

$$I_s \approx \frac{\sigma V_{s,r}}{R_r'} \, \frac{\omega_e}{\omega_e} = \frac{P\omega_\sigma V_{s,r}}{R_r' \omega_e}$$

$$\Rightarrow \omega_\sigma = \left(\frac{R_r'}{P V_{s,r}}\right) I_s \cdot \omega_e$$

$\Rightarrow$ ω_σ increases linearly with ω_e for a given current

③ T_{el}

$$T_{el} = \frac{3 V_{s,r}^2}{\omega_f \left[\left(R_s + \frac{R_r'}{\sigma} \right)^2 + X_{eq}^2 \right]} \cdot \frac{R_r'}{\sigma}$$

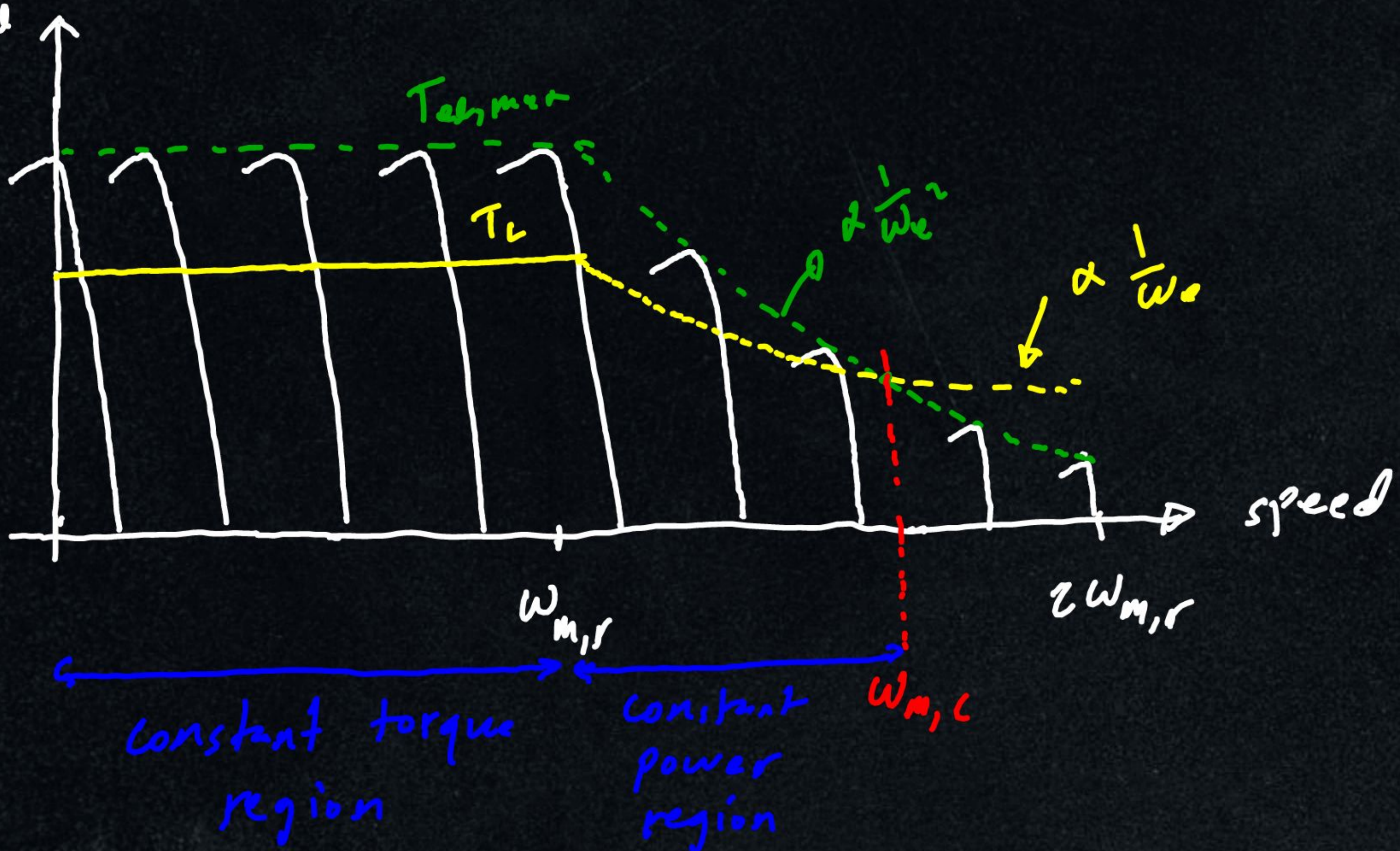
since σ is small

$$T_{el} \approx \frac{3 V_{s,r}^2 \, \sigma}{\omega_f \, R_r'} \, \frac{\omega_t}{\omega_f} = \frac{3 V_{s,r}^2 \, \omega_\sigma}{\omega_f^2 \, R_r'} = \frac{3 p^2 V_{s,r}^2}{R_r'} \, \frac{\omega_\sigma}{\omega_e^2}$$

$$\Rightarrow T_{el} \propto \frac{1}{\omega_e}$$

$\Rightarrow$ The developed power by the motor is constant

$\Rightarrow$ Thus, for $\omega_e > \omega_{e,r}$ $\Rightarrow$ The scalar controller gives constant constant power operation.



In the constant power mode, the torque keeps decreasing inversely with speed. At critical speed, ω_{mc}, the maximum torque is reached. Any attempt to operate beyond this speed will stall the motor. This is the limit of constant power region.

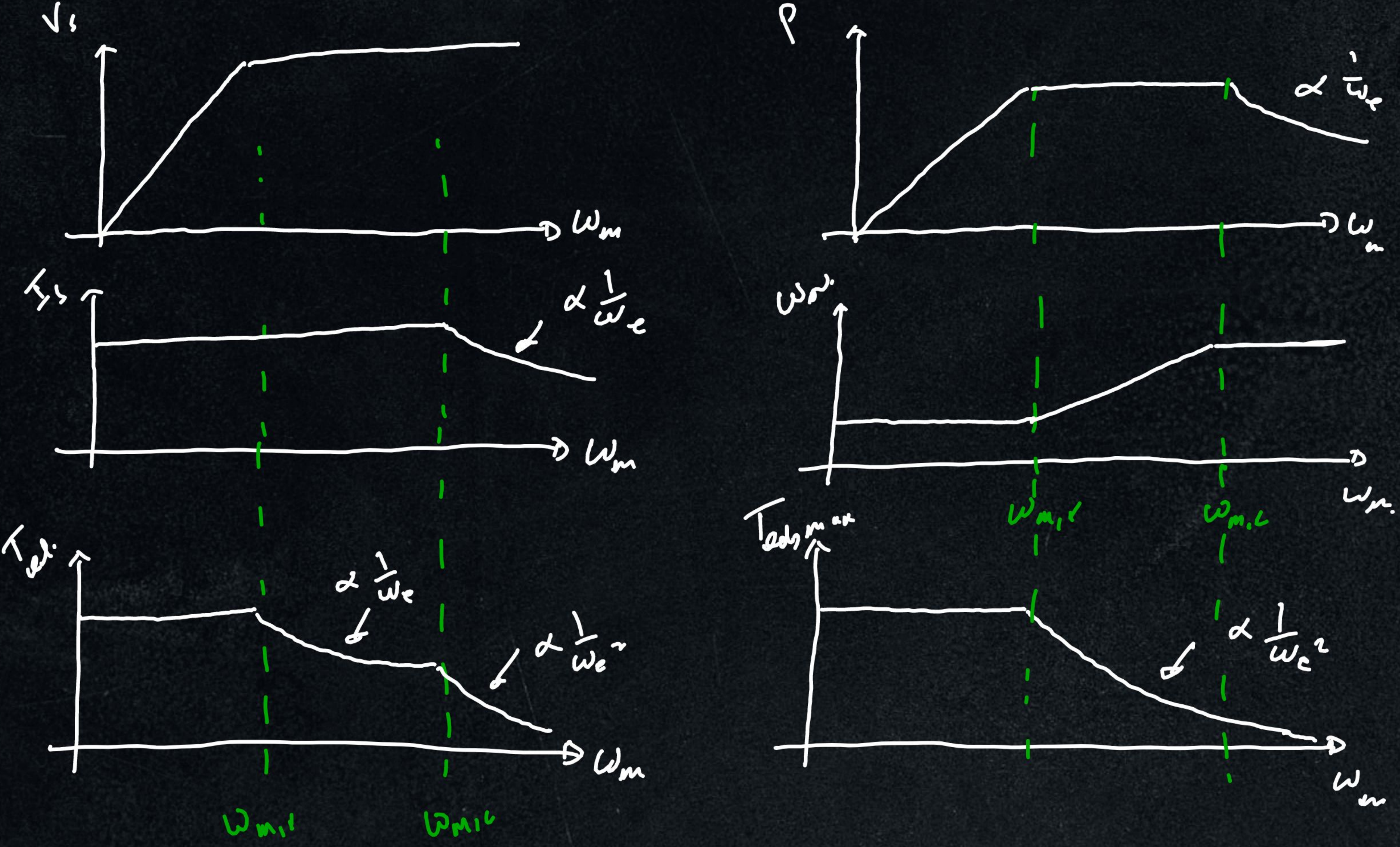
To prevent the torque from exceeding $T_{ed,max}$, the machine is operated at constant slip speed. In this case, the motor current reduces inversely with speed, and the torque decreases inversely at the speed square.

→ This characteristic is similar to series DC motor characteristic.

This must be constant

$$\omega_{sl} = \frac{R_r' I_s}{p V_{s,r}} \omega_e \quad , \quad T_{ed} = \frac{3p^2 V_{s,r}^2}{R_r'} \frac{\omega_{sl}}{\omega_e^2}$$

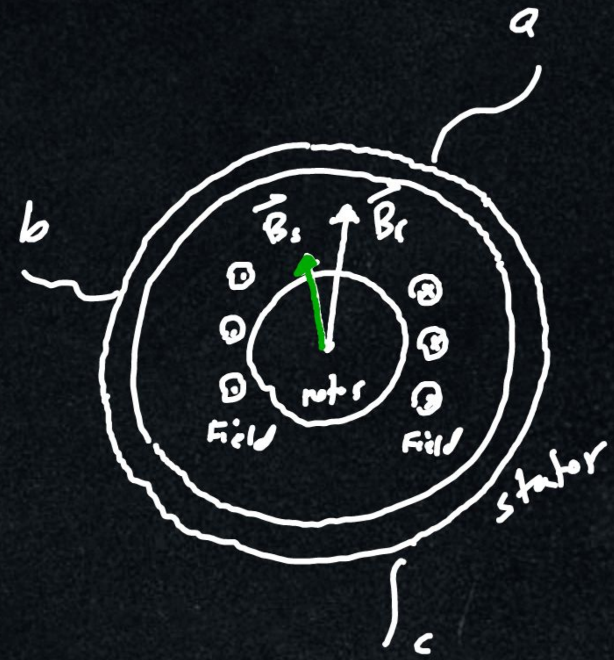
$$I_s \propto \frac{1}{\omega_e} \Rightarrow \omega_{sl} = \text{constant} \Rightarrow T_{ed} \propto \frac{1}{\omega_e^2}$$



V_s
ω_m
P
$\propto \frac{1}{\omega_e}$
ω_m
I_s
$\propto \frac{1}{\omega_e}$
ω_m
ω_{sl}
ω_m
T_{el}
$\propto \frac{1}{\omega_e}$
$\propto \frac{1}{\omega_e^2}$
ω_m
T_{edsmax}
$\propto \frac{1}{\omega_e^2}$
ω_m
$\omega_{m,1}$
$\omega_{m,2}$

Synchronous Machine Drive

operating principle of SM



- The field current produces $\vec{B}_r$.
- A set of 3ϕ voltages is applied to the stator, which produces a 3ϕ currents to follow in the stator windings.
- The currents produce a uniform rotating magnetic field, $\vec{B}_s$ which rotates at ω_s.
- The rotor is rotating by some external means at start to obtain a magnetic locking between the stator and rotor poles, and then produce a continuous induced torque ($T_{ind} = k\vec{B}_r \times \vec{B}_s$)

Classifications of SM :-

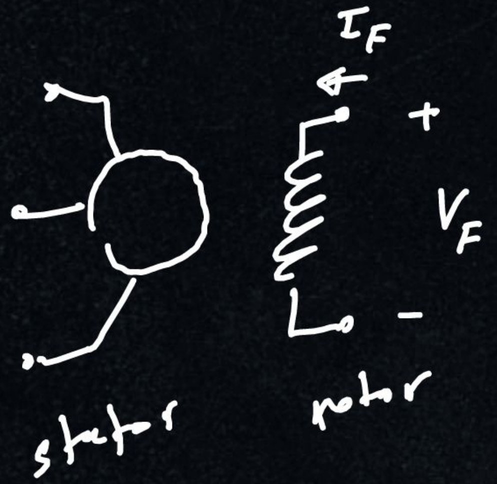
1) Wound field cylinderical

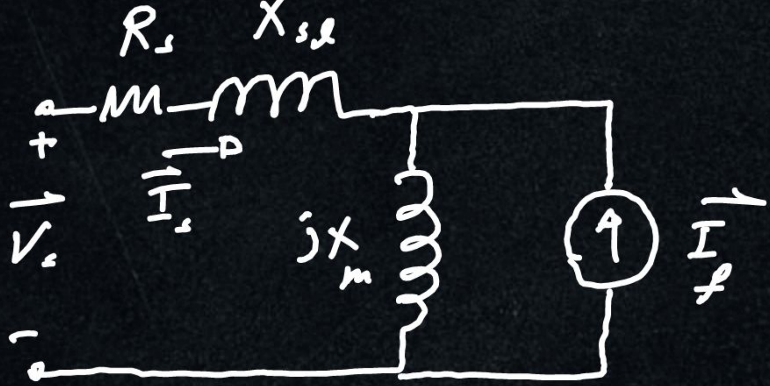
2) permanent Magnet

3) Wound field salient - pole.

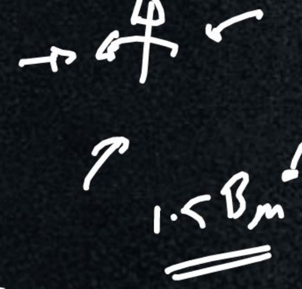
4) Reluctance

1) Wound Field Cylinderical Rotor



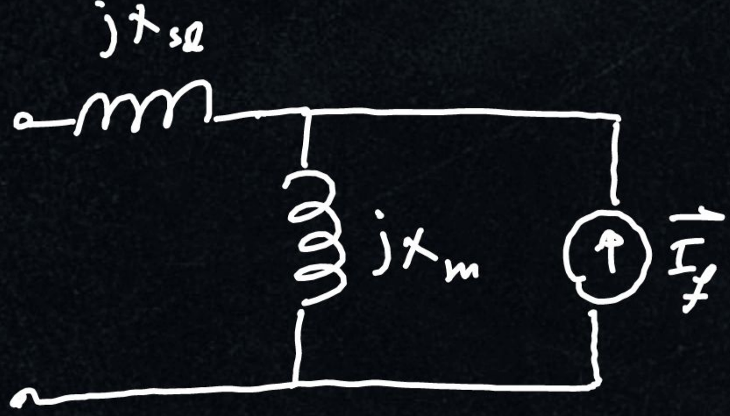


per-phase equivalent circuit



- The peak of rotating mmf produced by the rms current, I_f flowing through n_a turns is $\mathcal{F}_1 = \frac{3}{2} n_a (\sqrt{2}\, I_f)$
- The peak of rotating mmf produced by a dc current I_F flowing through n_f turns is, $\mathcal{F}_2 = n_f I_F$

$$\mathcal{F}_1 = \mathcal{F}_2 \implies I_f = n I_F \quad \text{where} \quad n = \frac{\sqrt{2}}{3} \frac{n_f}{n_a}.$$



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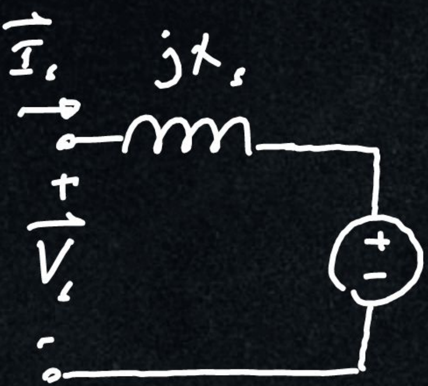


$$\vec{V}_f = jX_m \vec{I}_f = V_f \angle -\delta$$

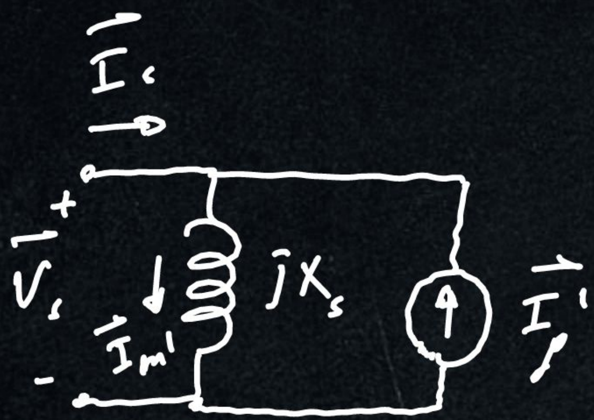
$$\vec{V}_f = V_f \angle -\delta$$

$$X_s = X_{s\ell} + X_m$$

$\downarrow$

synchronous reactance

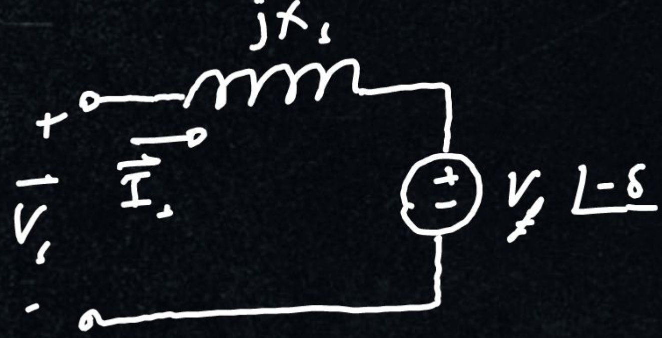
$\langle \equiv \rangle$

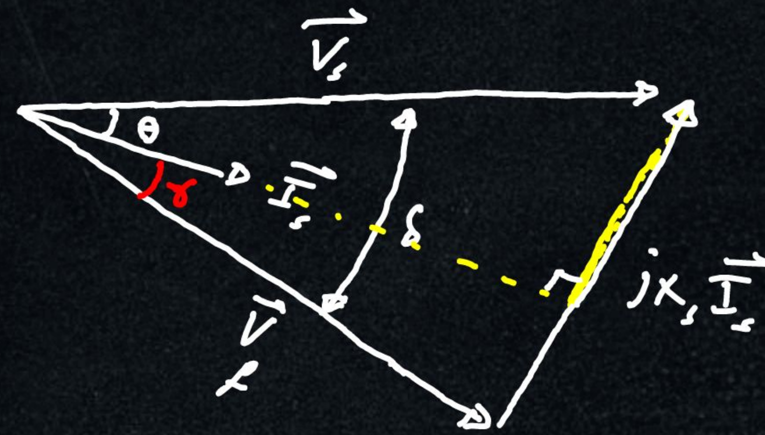


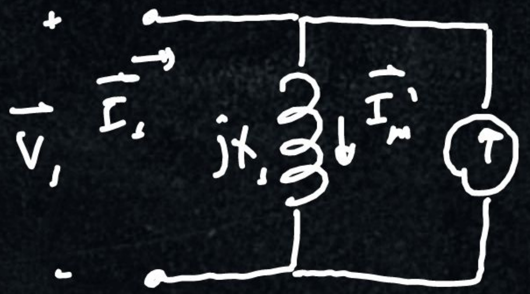
$$\vec{I}_f' = \frac{\vec{V}_f}{jX_s} = \frac{X_m}{X_s}\vec{I}_f = I_f' \angle -\left(\delta + \frac{\pi}{2}\right)$$

$$\vec{I}_m' = \vec{I}_s + \vec{I}_f' = \frac{\vec{V}_s}{jX_s}$$

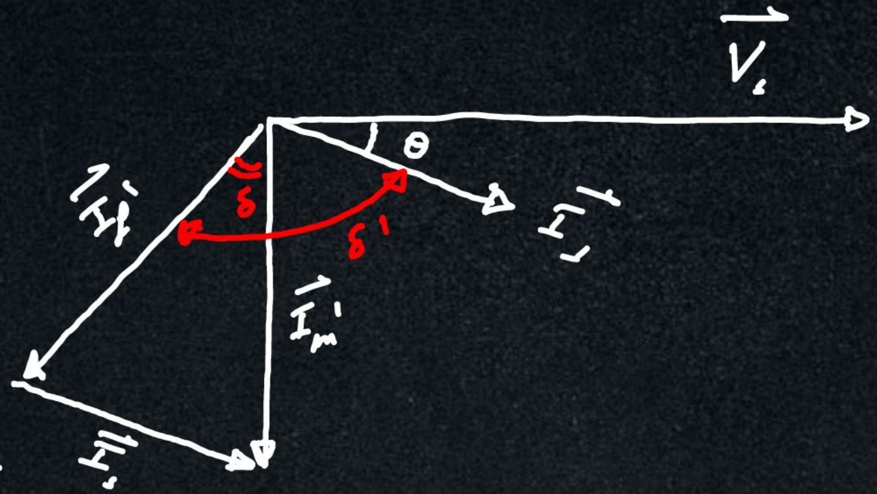
phasor diagram







$$\vec{I}_p' = I_p' \angle -(\delta + \frac{\pi}{2})$$



Torque Equations

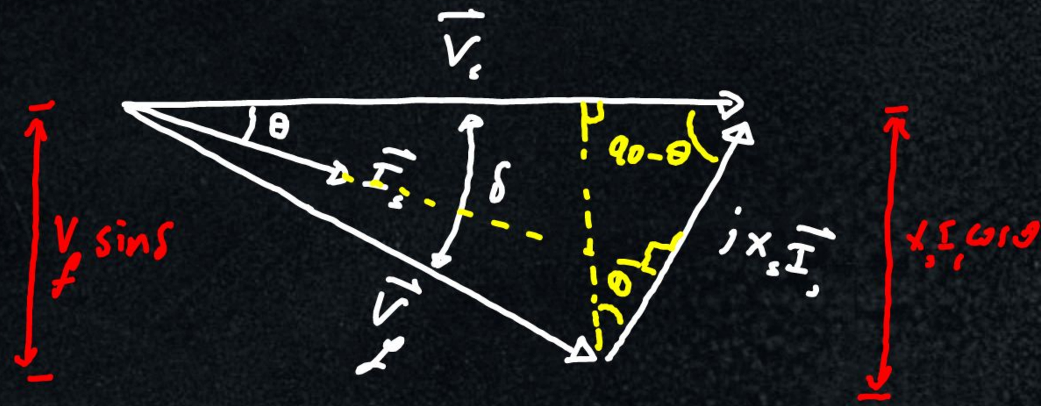
(1) $V_f \sin\delta = X_s I_s \cos\theta$

$$T_{el} = \frac{P_{in}}{\omega_s}$$

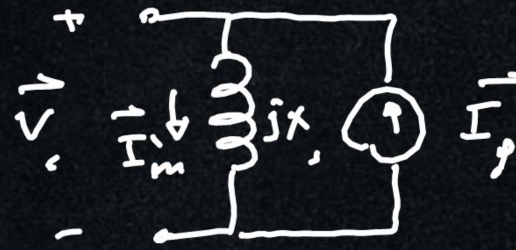
$$P_{in} = 3 V_s I_s \cos\theta$$

$$T_{el} = \frac{3 V_s I_s \cos\theta}{\omega_s}$$

$$\boxed{T_{el} = \frac{3 V_s V_f}{X_s \omega_s} \sin\delta}$$



② $T_{el} = \dfrac{3 V_s V_f}{\omega_s X_s} \sin\delta$



$\vec{I}_f{}' = \dfrac{\vec{V}_f}{X_s}$

$V_s = X_s I_m'$

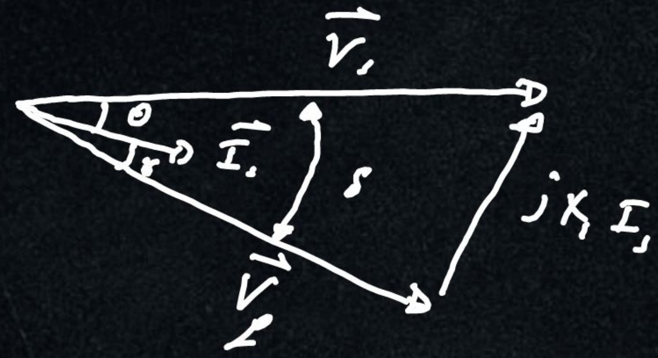
$V_f = X_s I_f'$

$$T_{el} = \frac{3 X_s^2 I_m' I_f'}{\omega_s X_s} \sin\delta = \frac{3 X_s I_m' I_f'}{\omega_s} \sin\delta = \frac{3 \omega_e L_s I_m' I_f'}{\left(\frac{\omega_e}{p}\right)} \sin\delta$$

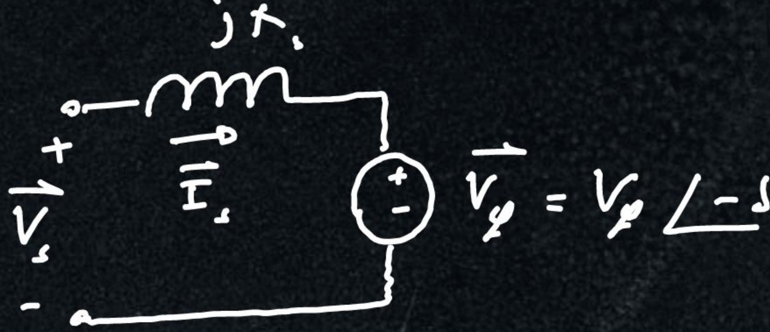
$$\boxed{T_{el} = 3 p L_s I_f' I_m' \sin\delta}$$

3

$$T_{el} = \frac{3 V_s I_s \cos\theta}{\omega_s} = \frac{P_{in}}{\omega_s}$$



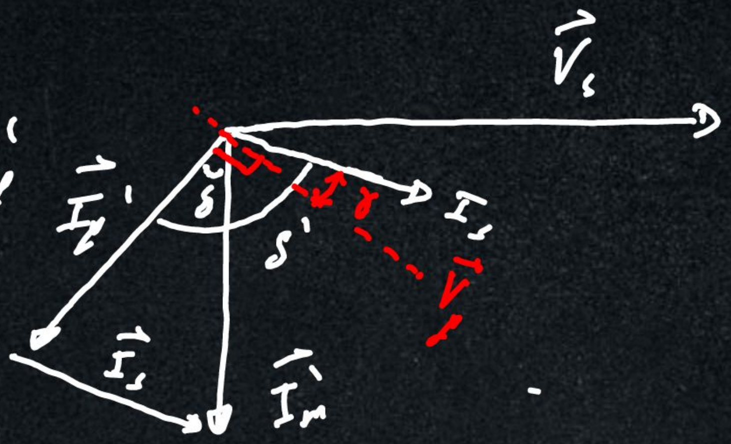
$$T_{el} = \frac{P_{air\text{-}gap}}{\omega_s} = \frac{3 V_f I_s \cos\gamma}{\omega_s}$$



$$\gamma + 90^\circ = \delta'$$

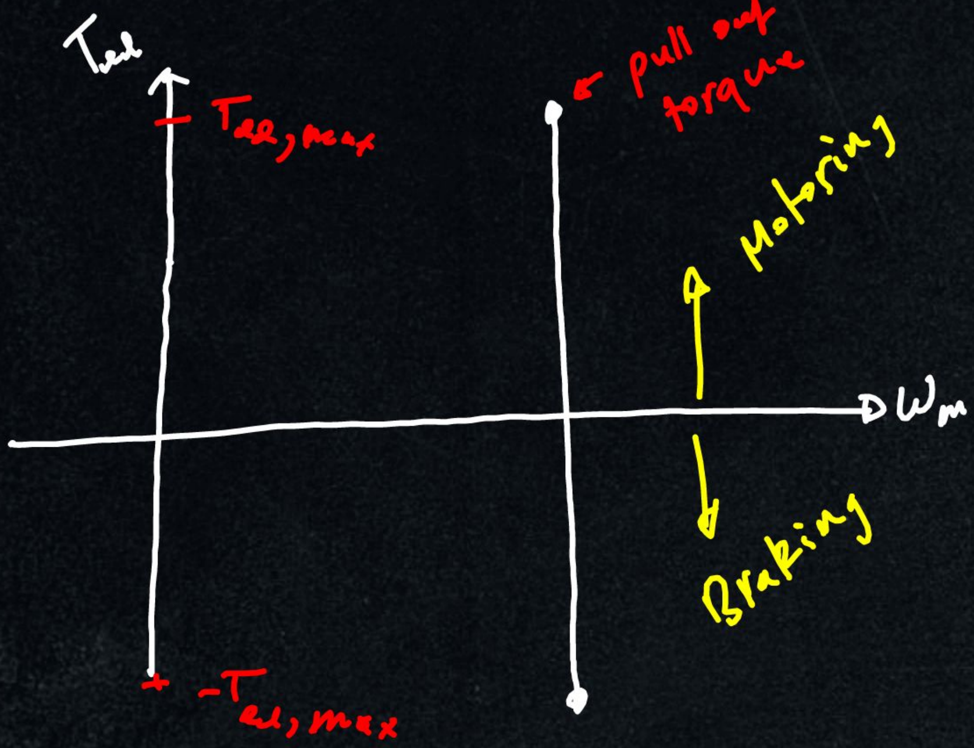
$$\gamma = \delta' - 90^\circ$$

$$T_{el} = \frac{3 V_f I_s \cos(\delta' - 90^\circ)}{\omega_s} \; ; \; V_f = X_s I_f'$$

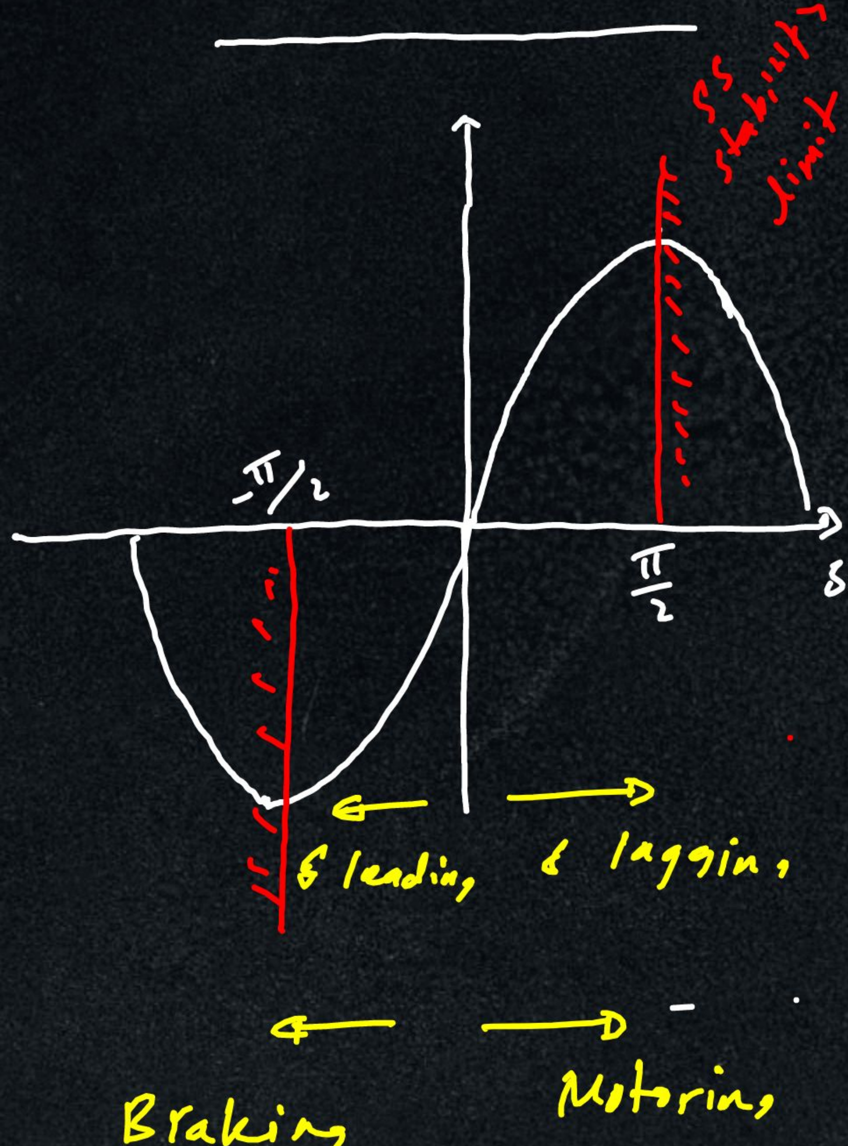


$$\boxed{T_{el} = 3 P L_s I_f' I_s \sin\delta'}$$

Torque-speed characteristic



Power curve



Variable Frequency Drive

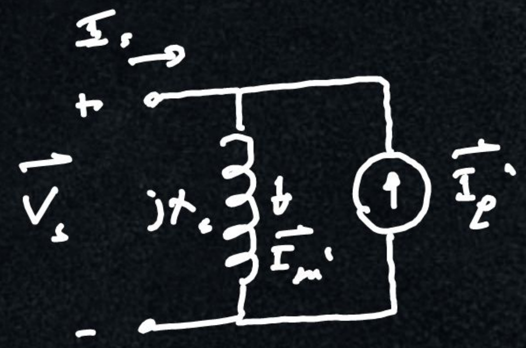
* The SM runs at fixed speed equal to ω_s; therefore, its speed can be controlled by the control of its supply frequency.

* With variable frequency control, the SM may operate in two modes:

  1) open loop V/f control
  2) closed loop V/f control

* The speed must be changed gradually to allow the rotor to track the changes in the revolving field speed.

- $\lambda_m = L_s I_m'$



$$I_m' = \frac{V_s}{X_s} = \frac{V_s}{\omega_e L_s} \propto \frac{V_s}{\omega_e}$$

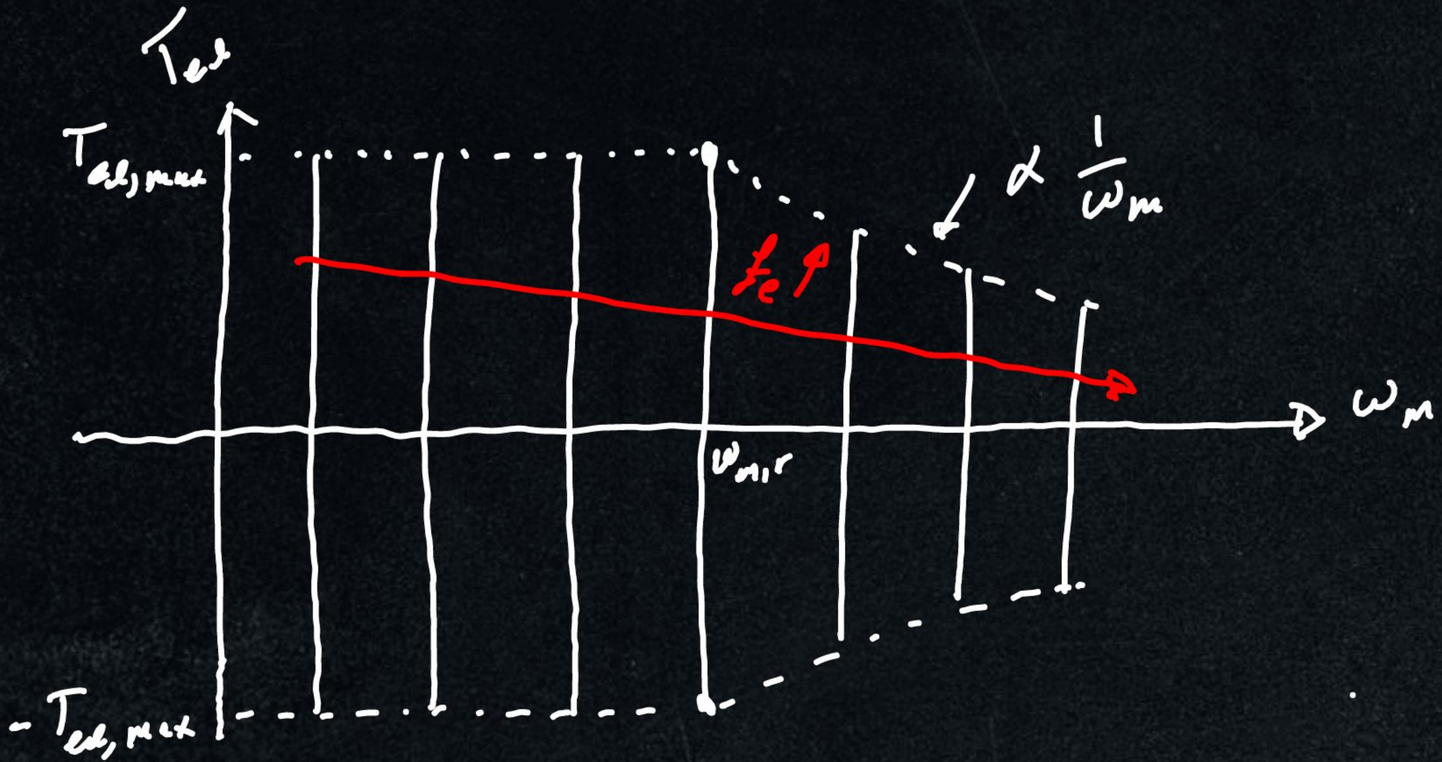
$$\lambda_m \propto \frac{V_s}{\omega_e}$$

$$\omega_m \leq \omega_{m,r} \Rightarrow \lambda_m = \lambda_{m,r}$$

$$\omega_m \geq \omega_{m,r} \Rightarrow \lambda_m = \lambda_{m,r} \frac{\omega_{m,r}}{\omega_m} \quad ; \quad \omega_m = \omega_f$$

- At low speed, V/f ratio is increased to compensate the voltage drop across R_s.

• Torque-speed characteristics of SM with variable frequency control.

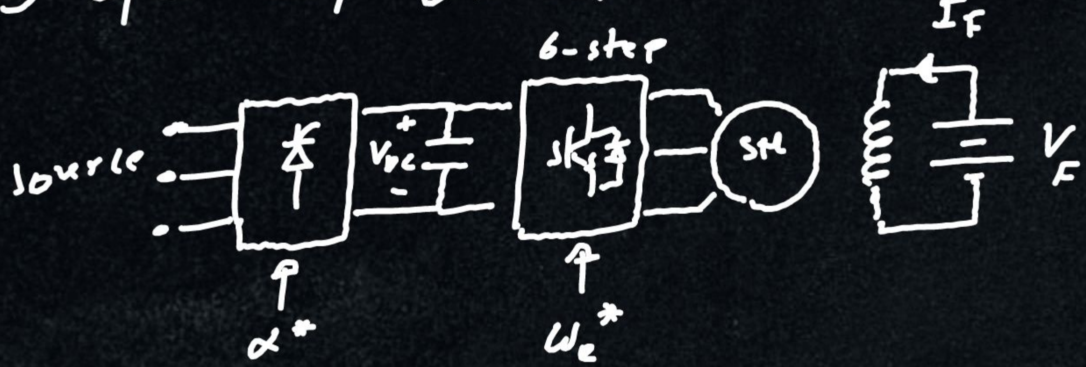


Drive circuits

1) 6-step inverter with controlled rectifier

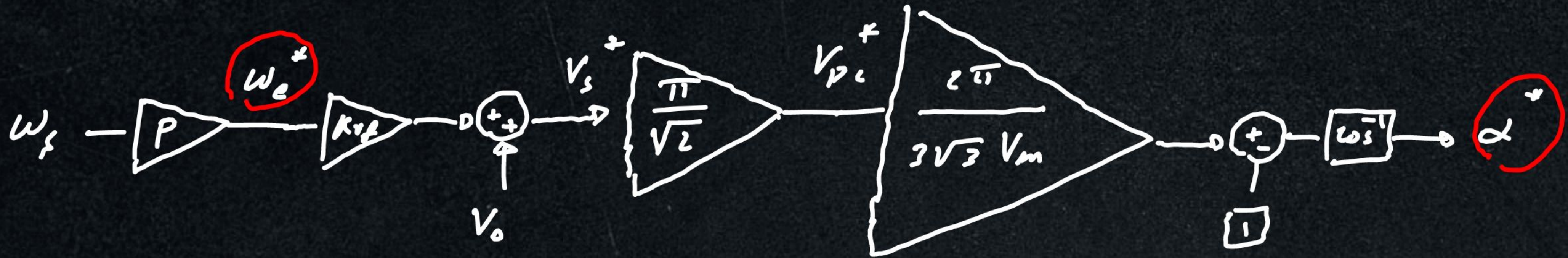
2) Three-phase inverter.

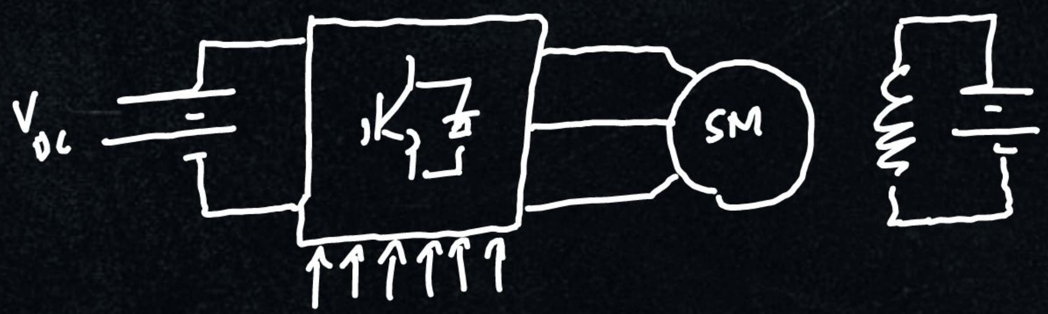
① open loop control

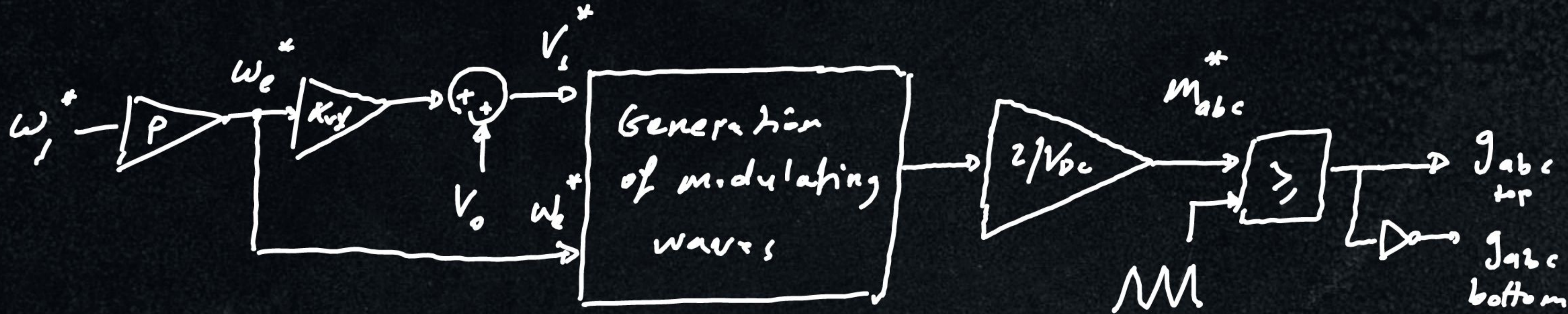


$$V_{DC} = \frac{3\sqrt{3}\, V_m}{2\pi}(1+\cos\alpha)$$

$$V_s = \frac{2}{\pi} V_{DC} \frac{1}{\sqrt{2}}$$







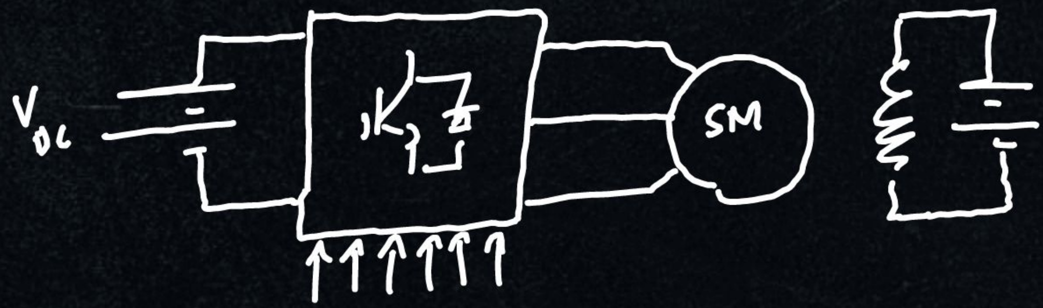
$$m_a^* = \sqrt{2}\, V_s \sin(\omega_e^* t)\, \frac{2}{V_{DC}}$$

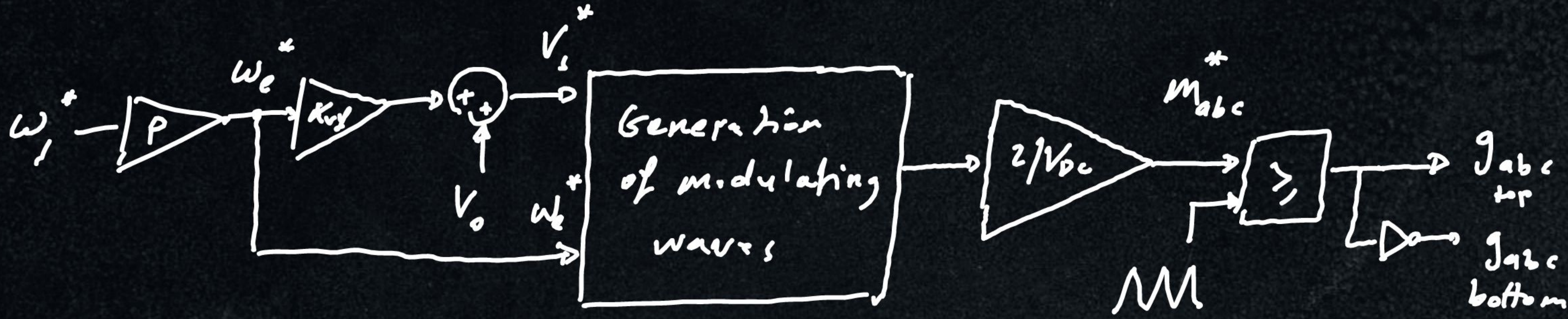
$$m_b^* = \sqrt{2}\, V_s \sin(\omega_e^* t - 2\pi/3)\, \frac{2}{V_{DC}}$$

$$m_c^* = \sqrt{2}\, V_s \sin(\omega_e^* t + 2\pi/3)\, \frac{2}{V_{DC}}$$

$$M = \frac{2\sqrt{2}\, V_s}{V_{DC}}$$

↳ modulation inden .





$$m_a^* = \sqrt{2}\, V_s \, \sin(\omega_e^* t) \, \frac{2}{V_{DC}}$$

$$m_b^* = \sqrt{2}\, V_s \, \sin(\omega_e^* t - 2\pi/3) \, \frac{2}{V_{DC}}$$

$$m_c^* = \sqrt{2}\, V_s \, \sin(\omega_e^* t + 2\pi/3) \, \frac{2}{V_{DC}}$$

$$M = \frac{2\sqrt{2}\, V_s}{V_{DC}}$$

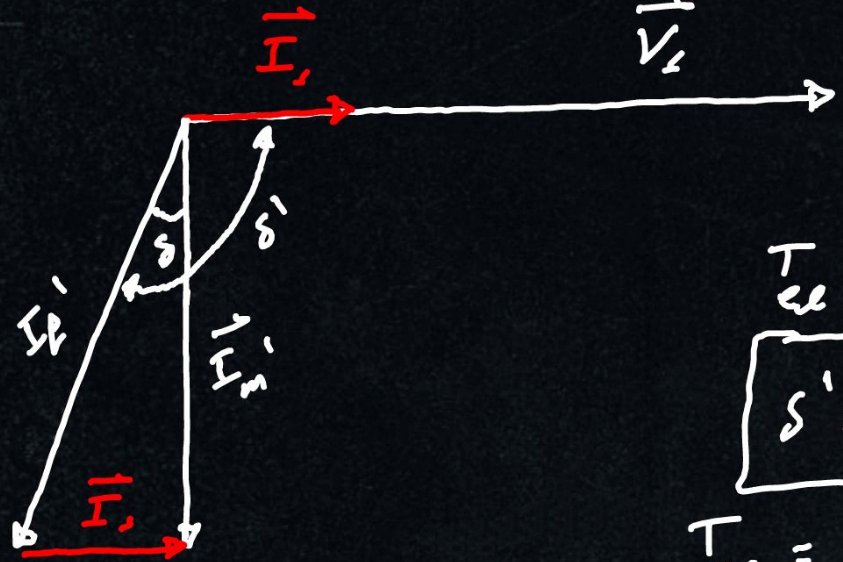
$\hookrightarrow$ modulation index.

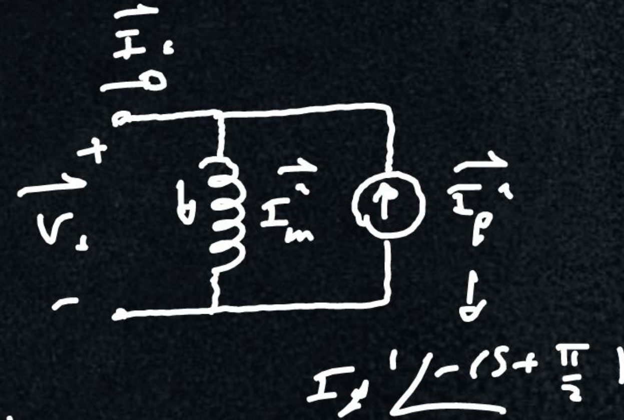
② Closed loop

Advantages

- Fast dynamic response for speed commands and load changes
- The power factor of wound field SM can be controlled by controlling its field current, I_F.

Unity PF operation





$$T_{el} = 3 p L_s I_f' I_s \sin\delta'$$

$$\boxed{\delta' = \delta + \frac{\pi}{2} \quad ; \quad \delta = \tan^{-1}\left(\frac{I_s}{I_m'}\right)}$$

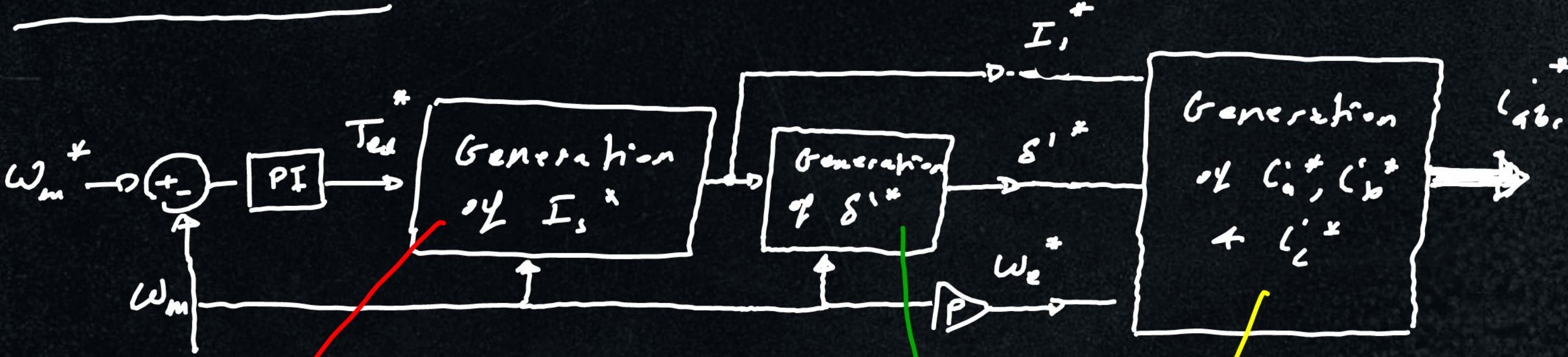
$$T_{el} = 3 p L_s I_f' I_s \cos\delta$$

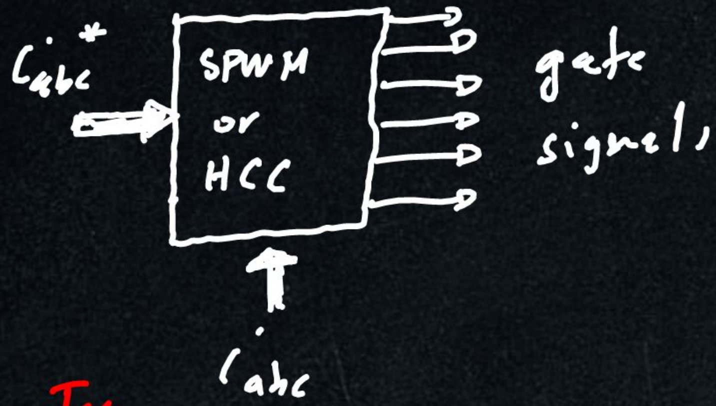
$$\boxed{T_{el} = 3 p L_s I_m' I_s}$$

$$\omega_m \leq \omega_{m,r} \Rightarrow I_m' = I_{m,r}'$$

$$\omega_m \geq \omega_{m,r} \Rightarrow I_m' = I_{m,r}' \frac{\omega_{m,r}}{\omega_m}$$

Control system





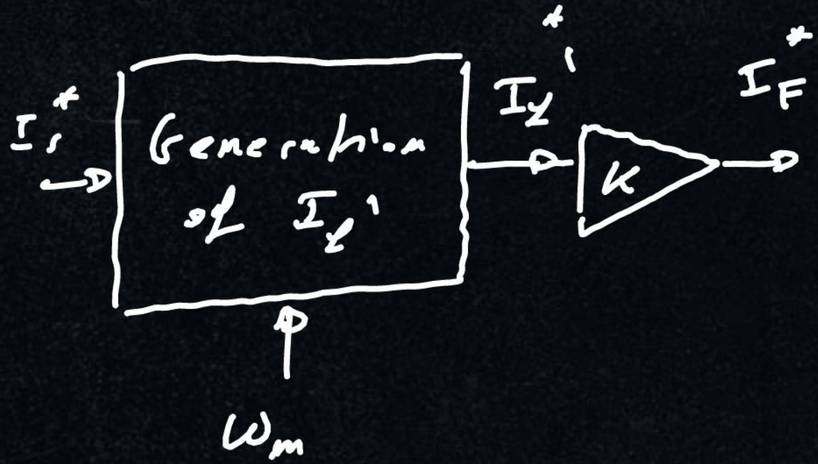
$$\omega_m \le \omega_{m,r} \Rightarrow I_s^* = \frac{T_{el}}{3pL_s I'_{m,r}}$$

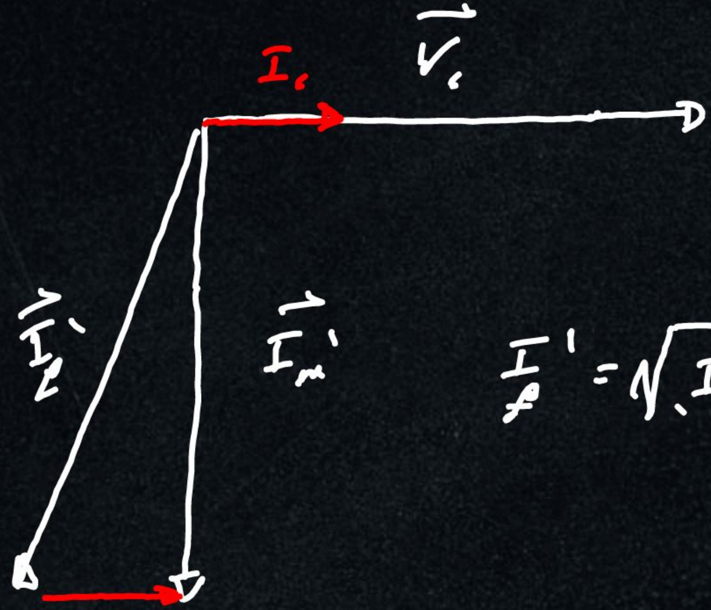
$$\omega_m \ge \omega_{m,r} \Rightarrow I_s^* = \frac{T_{el} \cdot \omega_m}{3pL_s I_{m,r}\, \omega_{m,r}}$$

$$\omega_m \le \omega_{m,r} \Rightarrow \delta' = 90^\circ + \tan^{-1}\left(\frac{I_s}{I'_{m,r}}\right)$$

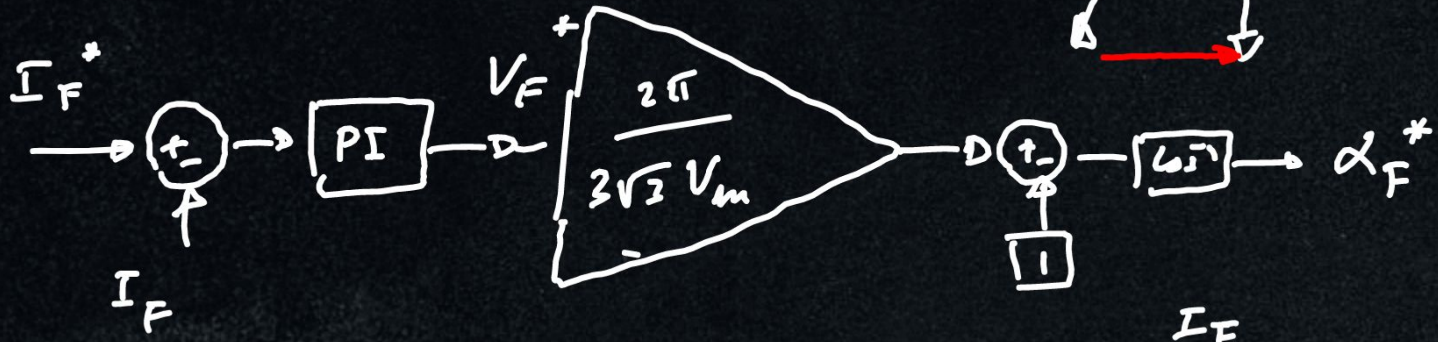
$$\omega_m \ge \omega_{m,r} \Rightarrow \delta' = 90^\circ + \tan^{-1}\left(\frac{I_s\, \omega_m}{I'_{m,r}\, \omega_{m,r}}\right)$$

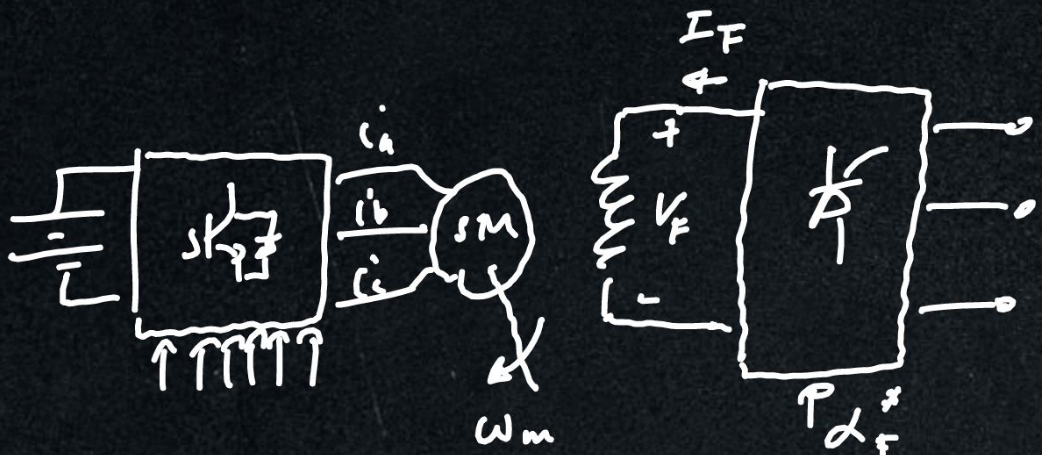
$$i_a'^* = \sqrt{2}\, I_s^* \sin(\omega_e^* t + \delta'^*)$$





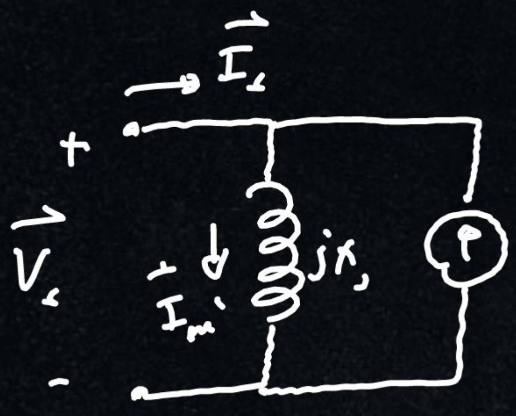
$$I_f' = \sqrt{I_m'^2 + I_s'^2}$$





2) Permanent Magnet Synchronous Motor (PMSM)

- Equivalent circuit



$\vec{I}_f' = I_f' \angle -(\delta + 90^\circ)$

$\underbrace{I_f'}_{\text{Constant}}$ ↳ Constant.

- The use of PM for excitation eleminates brushes + sliprings, and the associated maintenance. It also eleminates the field copper losses and the need for a DC source.

- Because of constant field current, the PF can not be controlled. If the field is designed to obtain a unity PF at full load, the motor operates at very low PF (leading) at light loads, resulting in poor efficiency.

- PMSMs are expensive because of high cost of magnets and rotor assembly.

Drive circuits

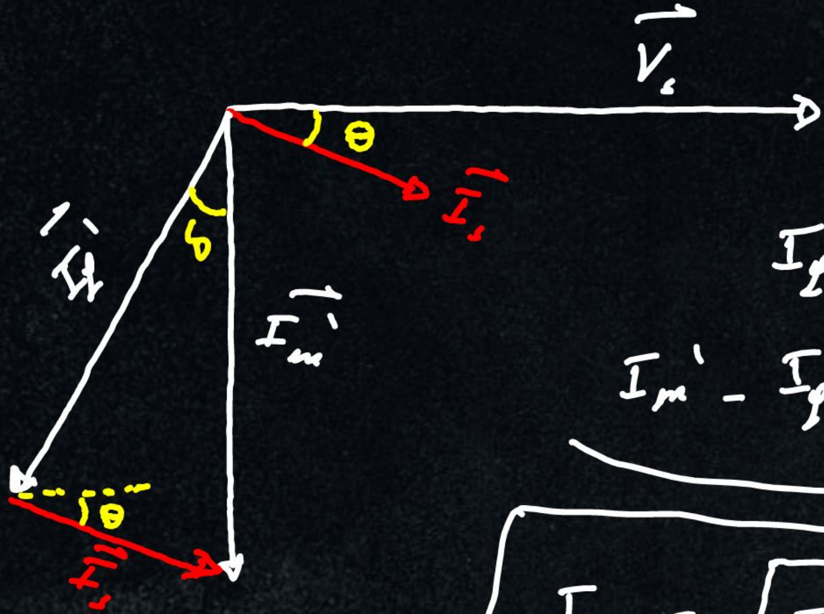
1) 6-step inverter with controlled rectifier

2) Three-phase inverter

① open loop control "similar to wound field SM"

② Closed loop control

$I_\phi' =$ constant $\Rightarrow$ PF is uncontrolled



$$I_\phi' \sin\delta = I_s \cos\theta$$

$$I_m' - I_\phi' \cos\delta = I_s \sin\theta$$

$$\boxed{I_s = \sqrt{I_\phi'^2 + I_m'^2 - 2 I_m' I_\phi' \cos\delta}}$$

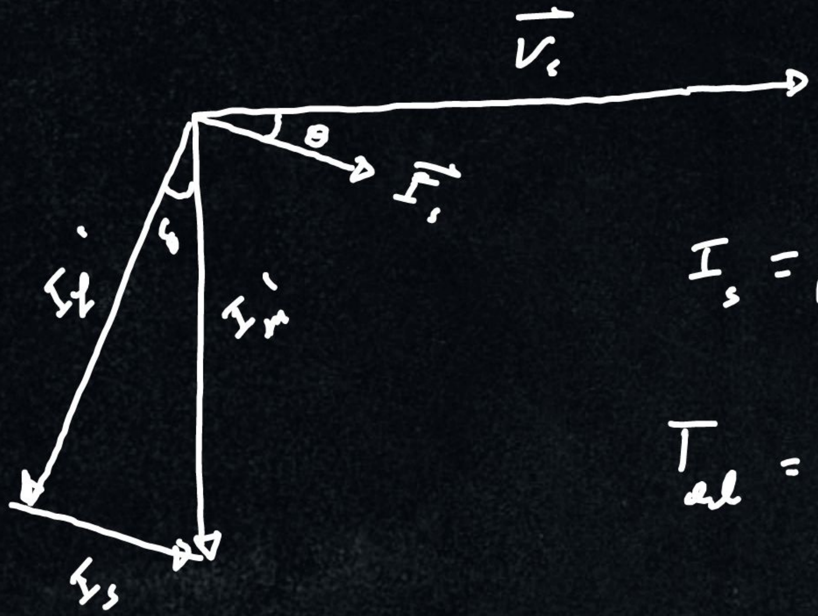
$$\boxed{T_{el} = 3 P l_s I_\phi' I_m' \sin\delta}$$

$\omega_m \leq \omega_{m,r} \Rightarrow I_m' = I_{m,r}'$

$\omega_m \geq \omega_{m,r} \Rightarrow I_m' = I_{m,r}\, \omega_{m,r} / \omega_m$

② closed loop control

$I_f' = \text{constant} \Rightarrow$ PF is uncontrolled



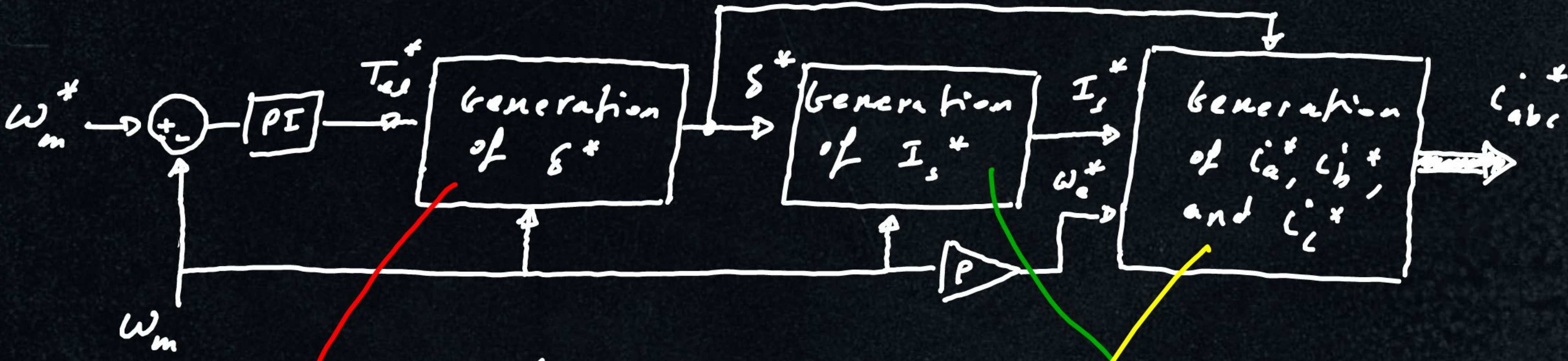
$$I_s = \sqrt{I_f'^2 + I_m'^2 - 2 I_f' I_m' \cos\delta}$$

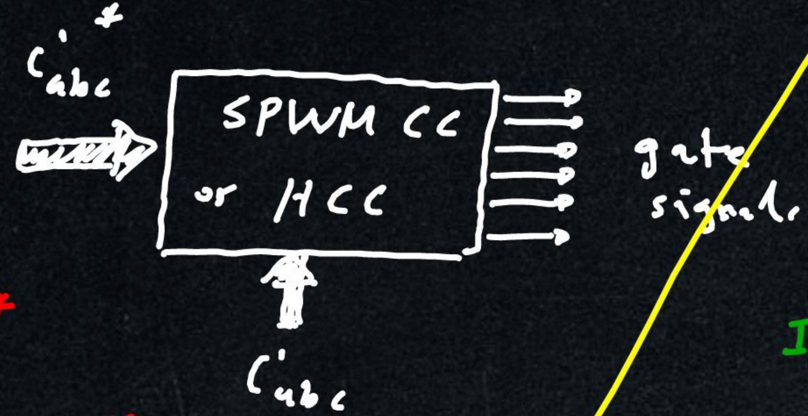
$$T_{el} = 3 p L_s I_f' I_m' \sin\delta$$

$$\omega_m \le \omega_{m,r} \Rightarrow I_m' = I_{m,r}'$$

$$\omega_m \geq \omega_{m,r} \Rightarrow I_m' = I_{m,r}' \frac{\omega_{m,r}}{\omega_m}$$

Control System





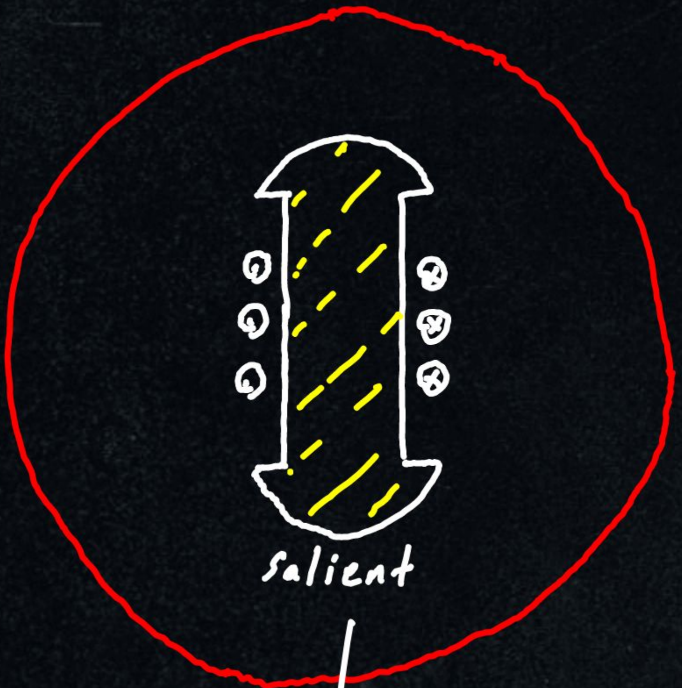
$$T_{ae}^* = 3 P L_s I_f' I_m' \sin\delta^*$$

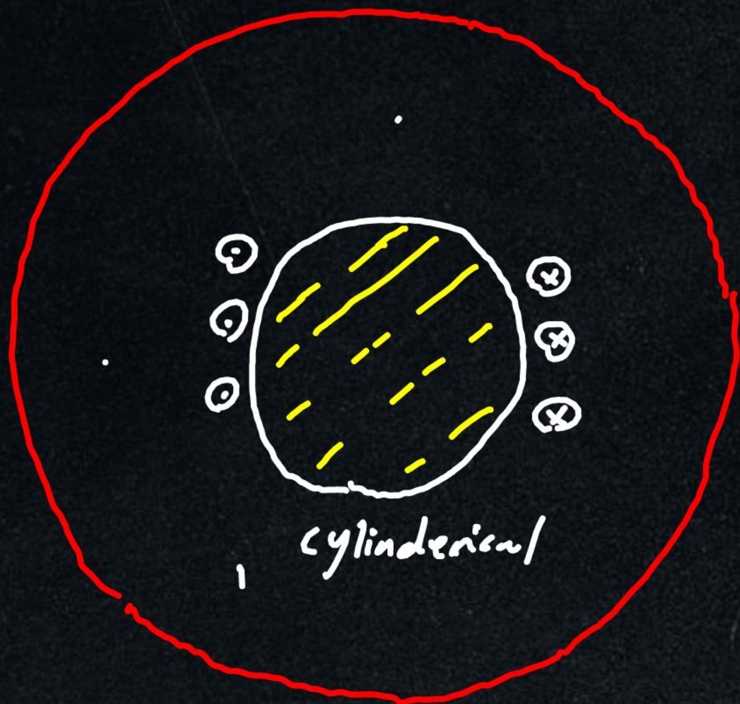
$$\delta^* = \sin^{-1}\left[\frac{T_{ae}^*}{3 P L_s I_f' I_m'}\right]$$

$$I_s^* = \sqrt{I_f'^2 + I_m'^2 - 2 I_f' I_m' \sin\delta^*}$$

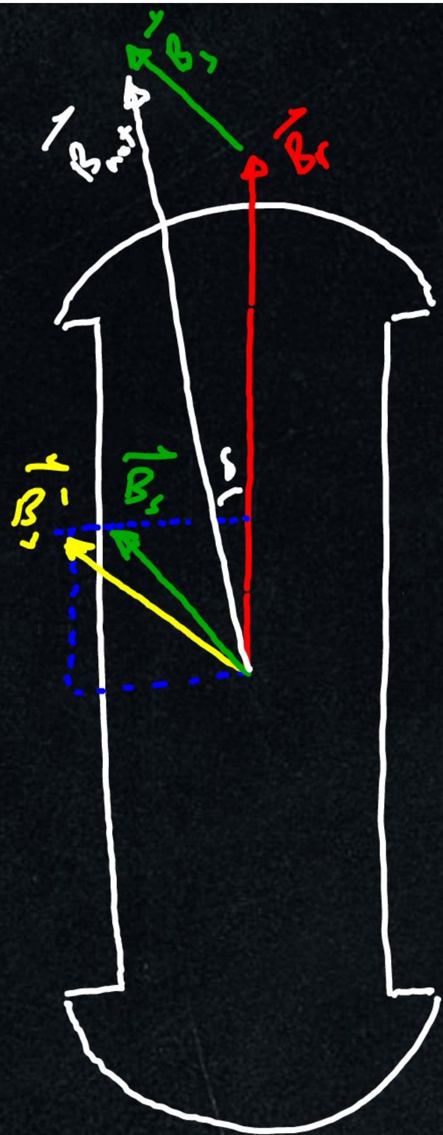
$$i_a^* = \sqrt{2}\, I_s \sin(\omega_e^* + \delta^*)$$

3) Wound field (salient pole) synchronous





It has field poles projecting out from the rotor core.



x y

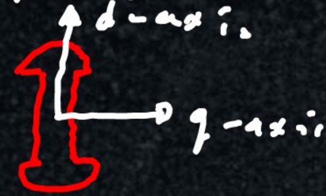
d q

$\vec{B}_s'$: stator field non-salient pole

$\vec{B}_s$: stator field salient poles

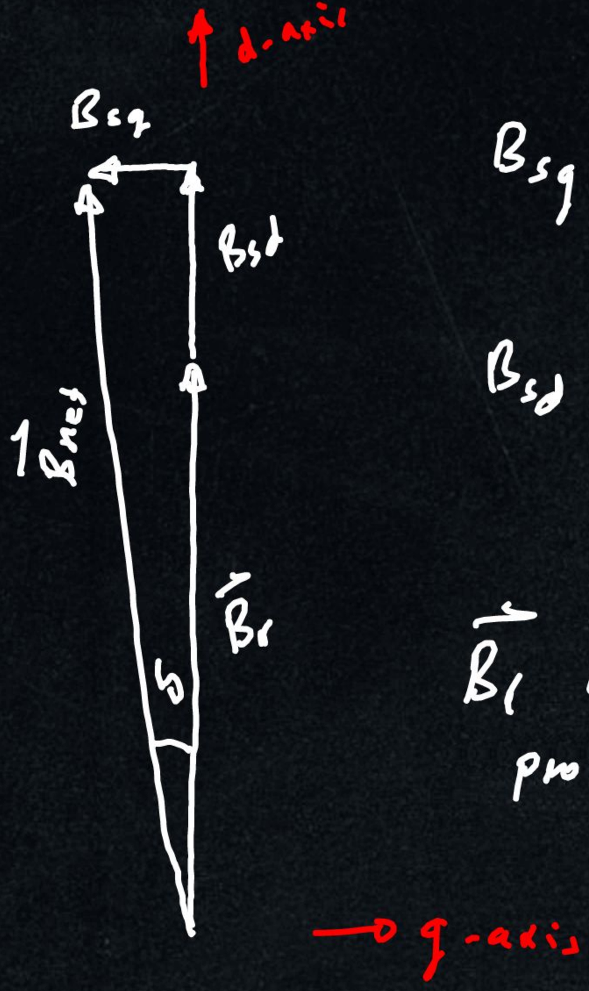
- The rotor magnetic field induces a voltage in the stator, which peaks in the wires directly under the pole faces.

- When a set of 3ϕ voltages is applied to the stator windings, a stator current will flow to produce a stator mmf, $\mathcal{F}_s$.



- $\mathcal{F}_s$ produces $\vec{B}_s$ (stator flux). However, the direct component of $\mathcal{F}_s$ produces more flux than the quadrature component since the reluctance at the direct-axis path is lower than the reluctance at the quadrature axis path.

$$\underbrace{\mathcal{F}}_{\text{mmf}} = \underbrace{\Phi}_{\text{flux}} \underbrace{\mathcal{R}}_{\text{reluctance}} \qquad \Phi_d > \Phi_q \Rightarrow B_{sd} > B_{sq}$$



B_{sq} produces the armature voltage V_q

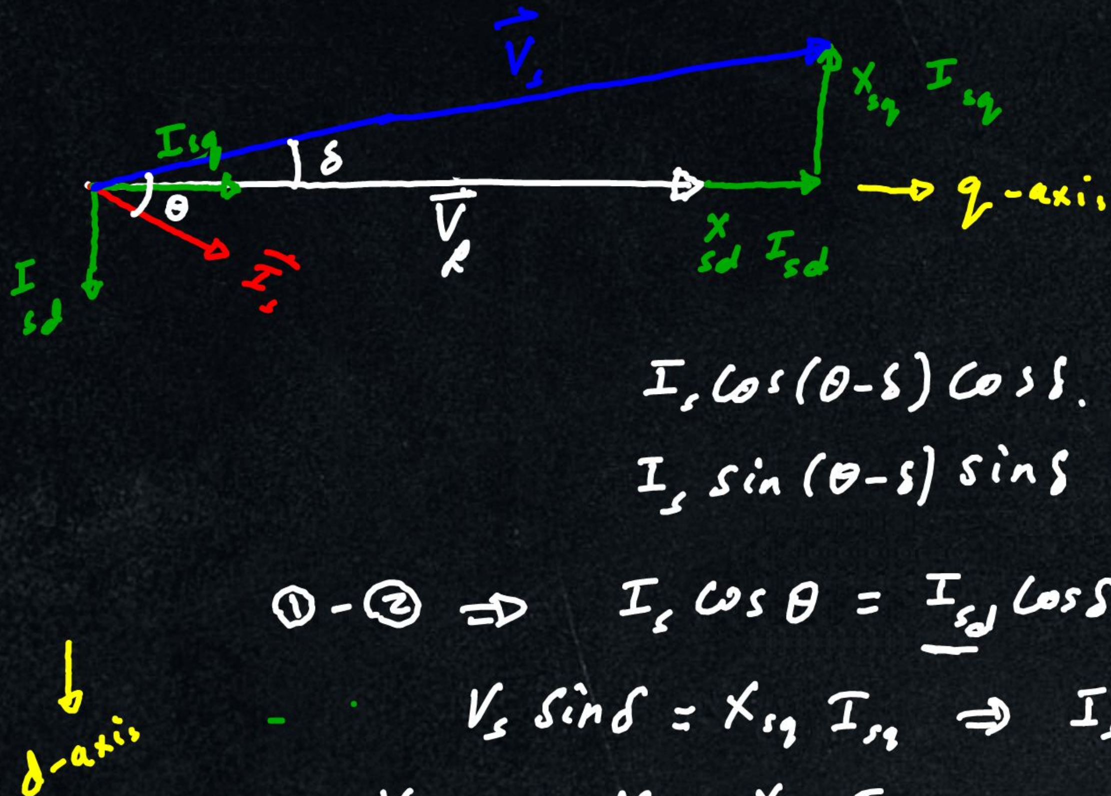
B_{sd} produces the armature voltage V_d

$\vec{B}_r$ interacts with $\vec{B}_{net}$ to produce T_{el} such that

$$T_{el} = K \vec{B}_r \times \vec{B}_{net} .$$

Phasor diagram and torque equation

$\cos A \cos B - \sin A \sin B$
$= \cos(A+B)$



$$I_s \cos(\theta-\delta) = I_{sd}$$

$$I_s \sin(\theta-\delta) = I_{sq}$$

$$I_s \cos(\theta-\delta)\cos\delta = I_{sd}\cos\delta \quad \text{----} \ (1)$$

$$I_s \sin(\theta-\delta)\sin\delta = I_{sd}\sin\delta \quad \text{----} \ (2)$$

$$(1)-(2) \Rightarrow \quad I_s \cos\theta = \underline{I_{sd}}\cos\delta - \underline{I_{sd}}\sin\delta \quad \text{----} \ (3)$$

$$V_s \sin\delta = X_{sq} I_{sq} \Rightarrow I_{sq} = \frac{V_s \sin\delta}{X_{sq}} \quad \text{---} \ (4)$$

$$V_s \cos\delta - V_f = X_{sd} I_{sd} \Rightarrow I_{sd} = \frac{V_s \cos\delta - V_f}{X_{sd}} \quad \text{---} \ (5)$$

4 + 5 in 3

$$I_s \cos\theta = \left(\frac{V_s \sin\delta}{X_{sq}}\right)\cos\delta - \left(\frac{V_s \cos\delta - V_f}{X_{sd}}\right)\sin\delta$$

$$= \frac{V_f}{X_{sd}}\sin\delta + \frac{V_s}{X_{sq}}\sin\delta\cos\delta - \frac{V_s}{X_{sd}}\sin\delta\cos\delta \qquad \Big/ \quad \sin 2\delta = 2\sin\delta\cos\delta$$

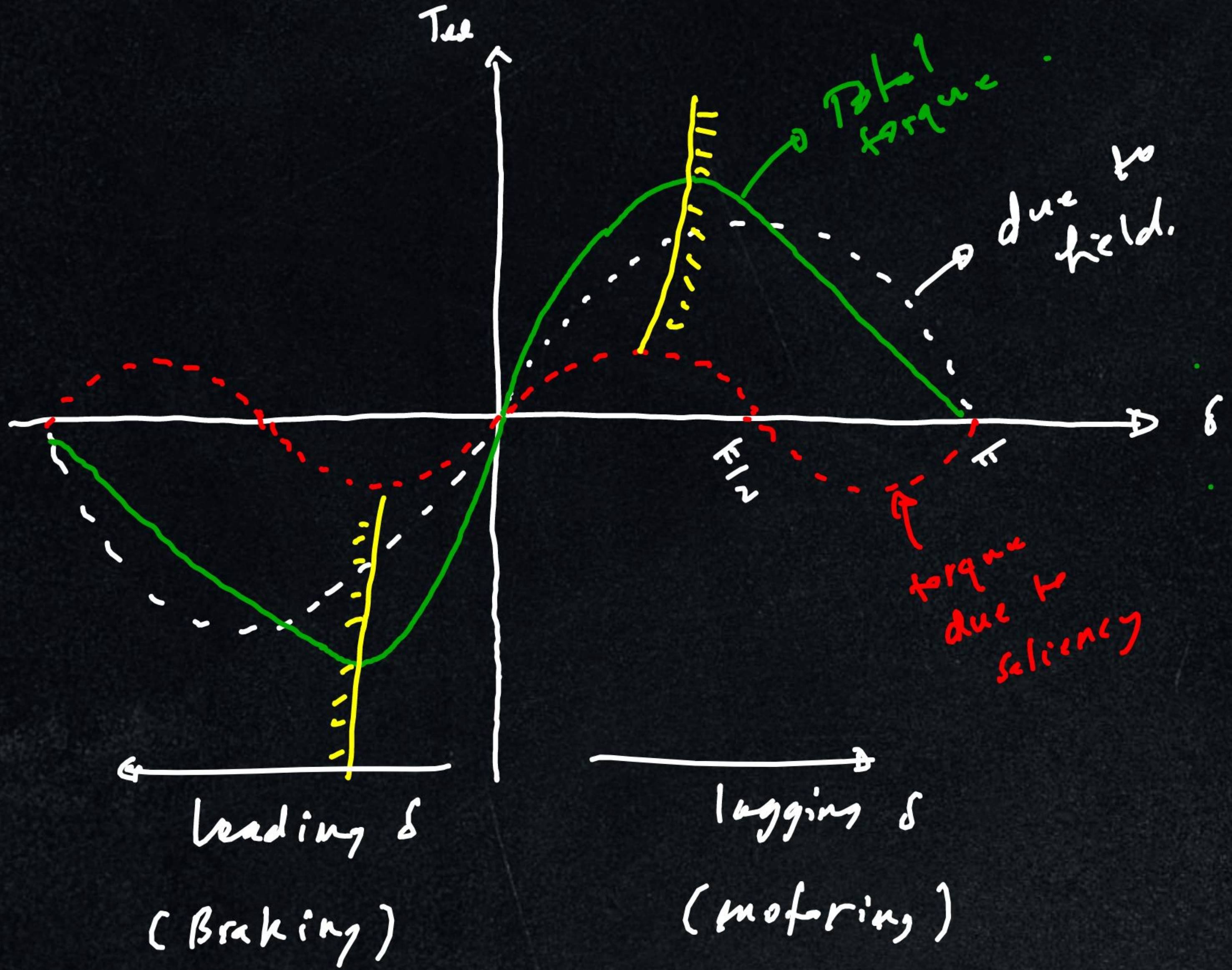
$$I_s \cos\theta = \frac{V_f}{X_{sd}}\sin\delta + \left(\frac{1}{X_{sq}} - \frac{1}{X_{sd}}\right)\frac{V_s}{2}\sin 2\delta$$

$$T_{dev} = \frac{P_{in}}{\omega_s} = \frac{3 V_s I_s \cos\theta}{\omega_s}$$

$$\Rightarrow \boxed{T_{dev} = \underbrace{\frac{3 V_s V_f}{X_{sd}\,\omega_s}\sin\delta}_{\text{synchronous torque}} + \underbrace{\frac{3}{2}\frac{V_s^2}{\omega_s}\left[\frac{1}{X_{sq}} - \frac{1}{X_{sd}}\right]\sin 2\delta}_{\text{reluctance torque}}}$$

synchronous torque: Dependent on field excitation

reluctance torque: indep. of field excitation.



Tel
Total torque
due to field.
δ
π/2
π
torque due to saliency
leading δ
(Braking)
lagging δ
(motoring)

4) Synchronous Reluctance Motor

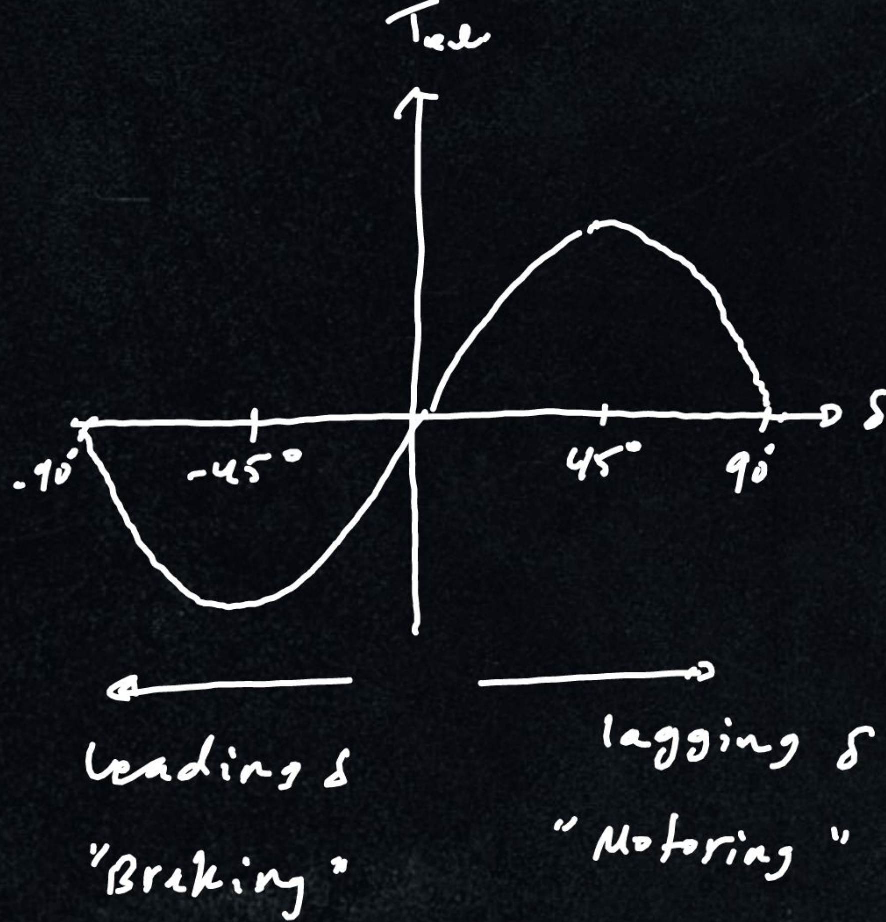
- It is similar to salient pole motor except that there is no field winding on the rotor.
- The armature circuit, which produces rotating magnetic field in the air-gap, induces a field in the rotor that has a tendency to align with the armature field.
- The reluctance motors are very simple and are used in applications where a number of motors are required to rotate in synchronizm.

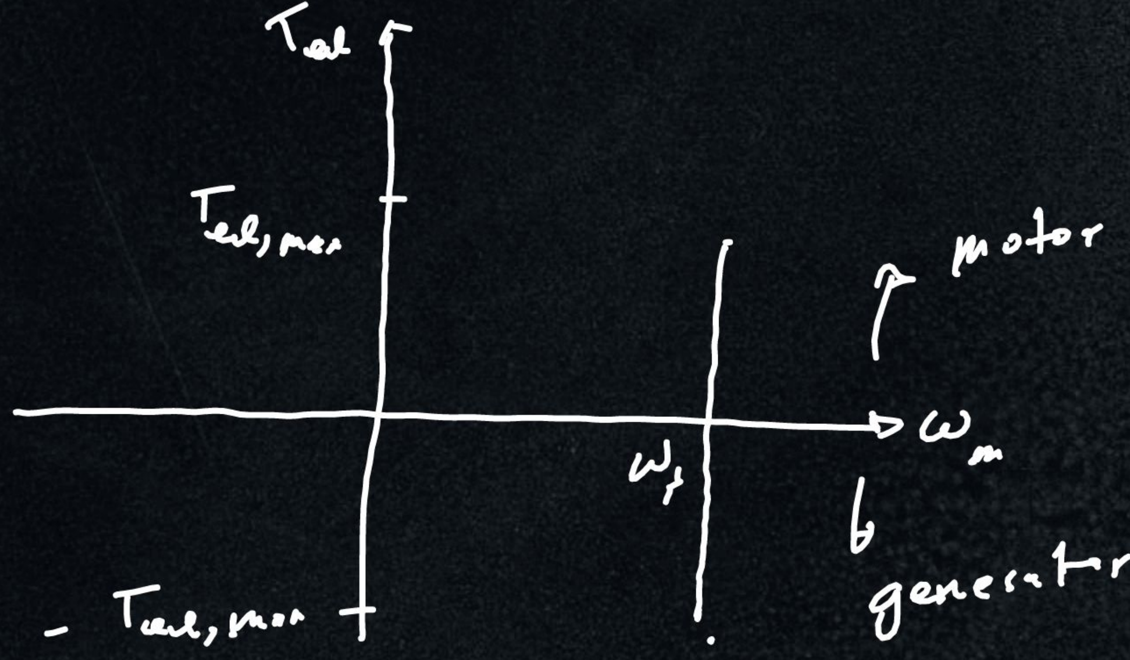
- Because of the absence of the field excitation, the entire I_m required to produce the air-gap flux has to come from the armature supply $\Rightarrow$ the machine has low lagging PF (0.65 - 0.75) at full load.

- The torque developed by the motor is :-

$$T_{el} = \frac{3}{2} V_1^2 \left[\frac{1}{X_{sq}} - \frac{1}{X_{sd}} \right] \sin 2\delta$$

- The pull out torque is reached at $\delta_{ss} = 45^\circ$


T_{el}
δ
$-90°$
$-45°$
$45°$
$90°$
leading δ
lagging δ
"Braking"
"Motoring"


T_{el}
$T_{el,max}$
$-T_{el,max}$
ω_m
ω_s
motor
generator